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AN INSTALLATION FOR MULTIPLEX-PULSE MODULATION

by C. J. H. A. STAAL.

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For radio-telephone communications very short waves are desired, because for one thing they can easily be beamed; this promotes secrecy and the transmitter can work on a lower power. With these very short waves the ordinary methods of modulation — amplitude and frequency modulation — cannot very well be used. What is required is pulse modulation, whereby pulses are time-modulated or width-modulated. Between these pulses others can be introduced the modulation of which can be employed for transmitting other conversations. This is termed multiplex-pulse modulation. An eight-channel installation working on this principle is described.

Introduction

In certain cases it is found very difficult and consequently highly expensive to lay a cable for a telephone or telegraph link between two places; we have in mind here, for instance, a link with an island or between two places in mountainous country. It may then be much more economical to have recourse to a radio connection which in its properties somewhat resembles a cable link and for which the French have a most typical name, "câble hertzien", (radio waves are sometimes also called Hertzian waves). For the properties of a radio link to resemble those of a cable link the whole of the electromagnetic energy of the transmitter has to be emitted as far as possible in the direction of the receiver, so that as little energy as possible is lost. Furthermore, just as in the case of a cable link, the necessary secrecy has to be ensured. The transmitters serving for such links are called "link transmitters" [1]). Now the beaming of electromagnetic waves is all the better according as the dimensions of the beaming device employed (e.g. parabolic mirror, horn, Yagi antenna) are greater with respect to the wavelength on which the link transmitter is working. Since the beaming device should preferably be kept as small as possible, for a good directional effect it is best to work on the shortest possible wavelength, viz. centimetric waves. But the use of very short waves has two consequences.

In the first place the application of a Hertzian cable is then limited to cases where the distance between the two places to be linked is not greater than the visual distance. Under certain conditions this may make it necessary to set up the aerials at a very high elevation.

In the second place, with centimetric waves the problem of modulation requires to be solved in a manner different from what is customary with longer waves, where amplitude or frequency modulation of a continuous, sinusoidal carrier is applied. As is known, centimetric waves are generated with the aid of magnetrons or velocity-modulation valves. It appears that these valves cannot very well be used for amplitude or frequency modulation, since it is very difficult to modulate in amplitude without frequency modulation arising at the same time, and vice versa. Moreover, the modulation characteristics are far from linear. When several conversations are being transmitted simultaneously, as is mostly required, a slight deviation from linearity is sufficient to cause troublesome cross-talk between the channels.

One must therefore employ other methods of modulation. With pulse modulation it is possible to overcome these difficulties, while still attaining a signal-to-noise ratio (determining the quality of the link) of the same order as reached with normal amplitude or frequency modulation. What is to be understood by pulse modulation will be made clear below.

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[1]) See for instance Philips Techn. Rev. 2, 171-176 and 229-306, 1937; 3, 59-64, 1938; 6, 120-127, 1941.

We have just mentioned the requirement that it must be possible to transmit several conversations simultaneously. Therefore, just as a single cable becomes, as it were, multicored thanks to the methods of carrier telephony, so it must also be possible for a "Hertzian cable" to be multi-cored.

We shall first begin by dealing briefly with the principles of pulse-modulation methods. One of these methods, so-called pulse time modulation, forms the basis for the system of multiplex pulse modulation which will be described here. We shall confine ourselves to a description of the modulator and of the demodulator. The high-frequency part of the transmitter and receiver will be left out of consideration here.

Principle of pulse modulation

When pulse modulation is applied the transmitter does not produce a continuous carrier (*fig. 1a*) but a sequence of high-frequency wave trains following each other at a certain interval of time (fig. 1*b*). The high-frequency voltage is then modulated by pulses. Typical of this modulation is the fact that both the amplitude and the frequency of the high-frequency alternating voltage are kept constant during the pulses.

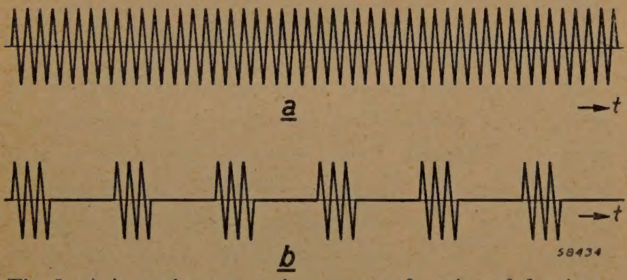

Fig. 1. *a*) A continuous carrier wave as function of the time *t*. *b*) A "carrier wave" consisting of wave trains in the form of rectangular pulses.

So long as the amplitude, width and interval of the pulses remain the same, the wave trains transmit only the repetition frequency of the pulses. To transmit a low-frequency signal one of the said factors is varied in the rhythm of this signal, or in other words the pulses are modulated with the low-frequency signal, which pulses in turn modulate the high-frequency carrier.

With this double modulation the series of pulses acts as a sort of auxiliary carrier. Since the high-frequency part (including the pulse modulation of the H.F. carrier) is beyond the scope of this discussion, the term pulse modulation will refer, throughout the remainder of the article, to the pulse modulation of the transmitted low-frequency signal. The form of the pulses need not in principle be rectangular, as represented in fig. 1, but in this article the pulses are understood to be rectangular.

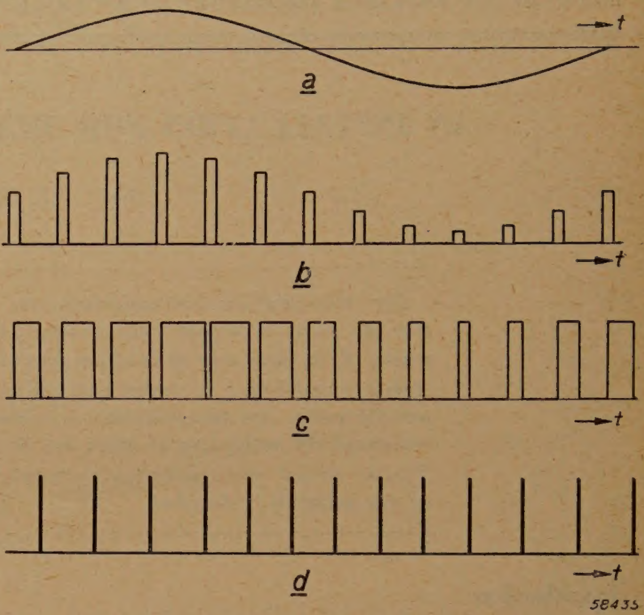

Fig. 2. Different methods of pulse modulation. In (*b*), (*c*) and (*d*) the amplitude, width or time, respectively, of the pulses is modulated in the rhythm of the low-frequency signal drawn in (*a*) as a sinusoidal curve. In (*d*) the pulses have been drawn very narrow for the sake of simplicity. (The time has been plotted on the horizontal axis.)

According as the amplitude of the pulses, their width or their position is varied in the rhythm of the low-frequency signal one speaks of pulse-amplitude, pulse-width or pulse time modulation (see *fig. 2*). For the reasons already indicated above, for very short waves pulse-amplitude modulation cannot be considered, because then also the amplitude of the high-frequency carrier is varied, and this is just what has to be avoided.

It is clear that in this way the low-frequency signal is not transmitted in its entirety but only in "pieces", though in a sufficiently large number to be able to build up again the whole signal at the reception end. Thus the continuous curve representing the amplitude of the signal as a function of time is transmitted in the form of a succession of dashes (see *fig. 3*), each dash corresponding to a certain pulse and its ordinate indicating the time (or width or amplitude) of the pulse.

To characterize a continuous curve unambiguously by dashes, these must lie sufficiently close together, and the less simple the curve the closer the dashes have to be. To put it more precisely, this means that the repetition frequency of the pulses must be great with respect to the highest signal frequency to be transmitted. It appears that good results can already be obtained when the said frequency ratio is 2.5 to 3. In a conversation the highest frequency to be transmitted is roughly equal to 3400 c/s. The repetition frequency must therefore be at least about 10,000 c/s.

Principle of multiplex-pulse modulation

If several conversations, say n in number, are to be transmitted simultaneously with pulse modulation then this is done, in principle, in the following way. A series of equal and equi-distant pulses are produced. Let us suppose that these pulses are numbered in their natural sequence. For the first conversation one modulates (in amplitude, width or time) only those pulses numbered $p + (n + 1), p + 2\ (n + 1), p + 3\ (n + 1)$, etc. ($p$ being a whole number), for the second conversation only the pulses $p + 1$, $p + 1 + (n + 1), p + 1 + 2\ (n + 1), p + 1 + 3\ (n + 1)$, etc, thus in general for the q-conversation ($1 \leqq q \leqq n$) only the pulses $p + q - 1, p + q - 1 + (n + 1), p + q - 1 + 2\ (n + 1), p + q - 1 + 3\ (n + 1)$. etc.

The pulses numbered $p + n, p + n + (n + 1), p + n + 2\ (n + 1), p + n + 3\ (n + 1)$ etc. are left unmodulated; they serve to indicate on the time axis the place where a new series of n impulses begins (belonging to the n different conversations) and are called synchronizing pulses.

Thus there corresponds to each conversation a specific series of equi-distant pulses with a repetition frequency $(n + 1)$ times as small as that of the original series of pulses.

If we draw a comparison with the known method of transmitting several conversations simultaneously as applied in carrier-telephony, it is seen that whereas in carrier-telephony the various "channels" (conversations) are displaced in frequency and laid side by side, with multiplex-pulse modulation they are displaced in time and laid side by side.

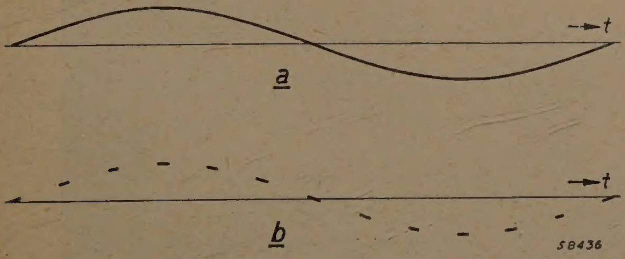

Fig. 3. With every method of pulse modulation a curve (a) representing a signal as a function of time is transmitted in the form of a sequence of dashes (b).

When in carrier-telephony more channels have to be transmitted then the total frequency band is made proportionally wider, whilst with multiplex-pulse modulation the total number of pulses per second, thus the repetition frequency, becomes proportionally greater. The fact is that on the one hand the minimum repetition frequency of pulses belonging to one conversation is fixed — as already stated, it is about 10,000 c/s — whilst on the other hand in the approximately 100 microseconds between two successive pulses belonging to one conversation a pulse of each of the other conversations has to be taken up.

In the case where, as with the system to be described below, one is working for instance with a total of nine channels, namely eight speech channels and a synchronizing channel ($n = 8$), the interval between two successive pulses is 11.1 μsec. and the repetition frequency of the total number of pulses 90 kc/s.

The technical realization of the principle of multiplex-pulse modulation outlined above depends, of course, upon the method of pulse modulation to be applied, but even with one particular method the execution may still vary considerably. In this article we shall direct our attention exclusively to the apparatus for multiplex-pulse modulation which has been developed by Philips and is illustrated in *fig. 4*.

The modulator

The method of pulse-modulation applied

The installation to be described here is based upon pulse-width modulation converted into pulse-time modulation, so that the actual transmission is in time-modulated pulses.

The unmodulated pulses are generated by causing a saw-tooth voltage (*fig. 5a*) to act upon the control grid of a valve. The valve will pass current through so long as the saw-tooth voltage is positive, that is to say in the first half of each saw-tooth cycle (starting to count the cycles from a vertical part of the saw-tooth line). Owing to the flow of grid current, a large portion of the positive part of the saw-tooth voltage appears across a series resistor, so that during this interval the grid voltage itself is limited. Consequently the anode current consists of a series of current pulses (fig. 5b) with the duration of each pulse amounting to half the saw-tooth cycle. When a low-frequency signal is applied to the same control grid (fig. 5c), then the point of intersection of the downward part of the saw-tooth on the horizontal time axis will be displaced to the left

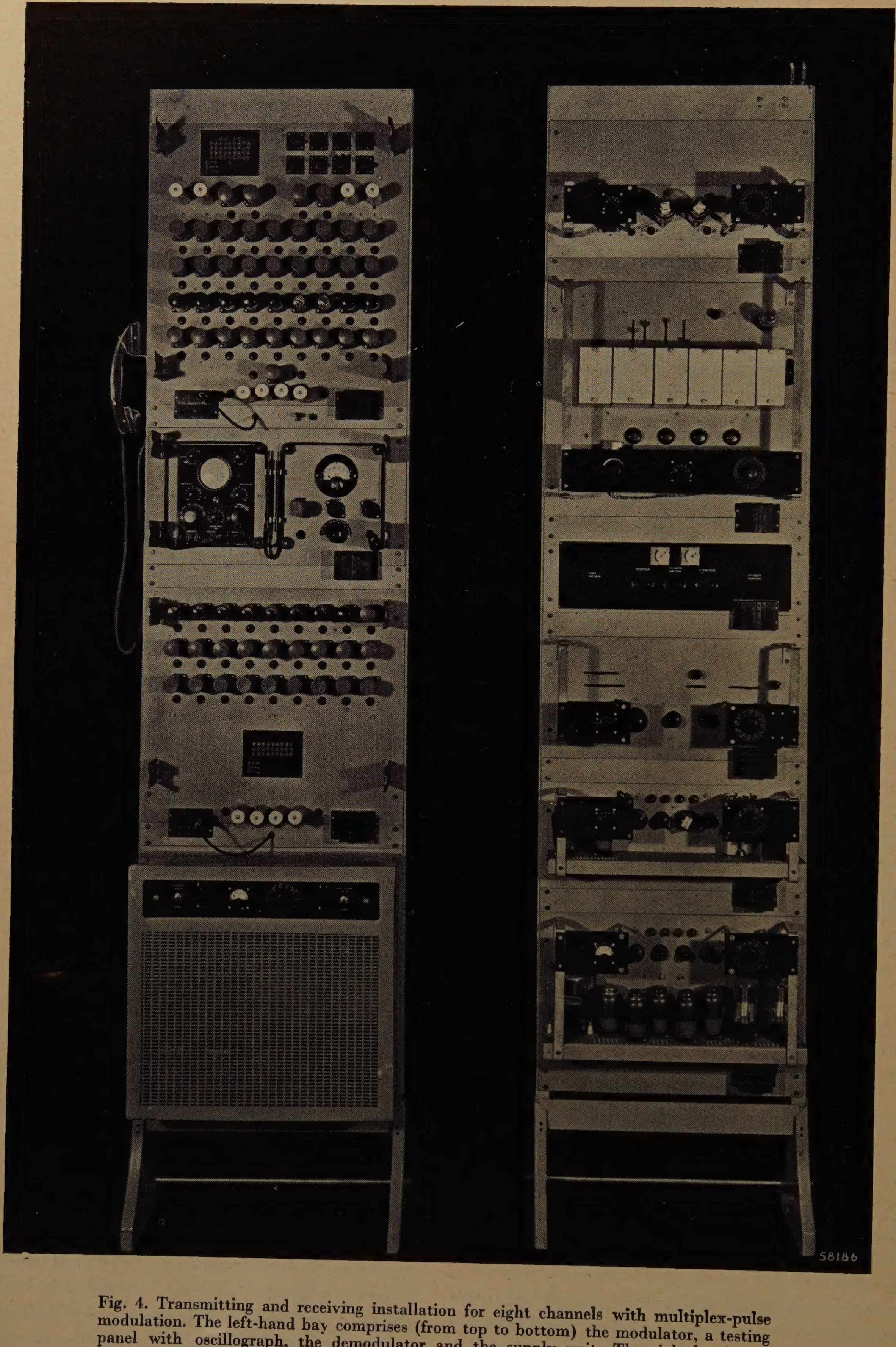

Fig. 4. Transmitting and receiving installation for eight channels with multiplex-pulse modulation. The left-hand bay comprises (from top to bottom) the modulator, a testing panel with oscillograph, the demodulator and the supply unit. The right-hand bay contains the high-frequency part of the apparatus.

or right over a distance proportional to the amplitude of the low-frequency signal; the point of intersection of the vertical part of the saw-tooth on the time axis is not displaced (fig. 5*d*). The valve will therefore interrupt the current somewhat earlier or somewhat later then when there is no low-frequency signal, so that the current pulses will be somewhat shorter or longer (fig. 5*e*) and one thus gets a width-modulation of the pulses.

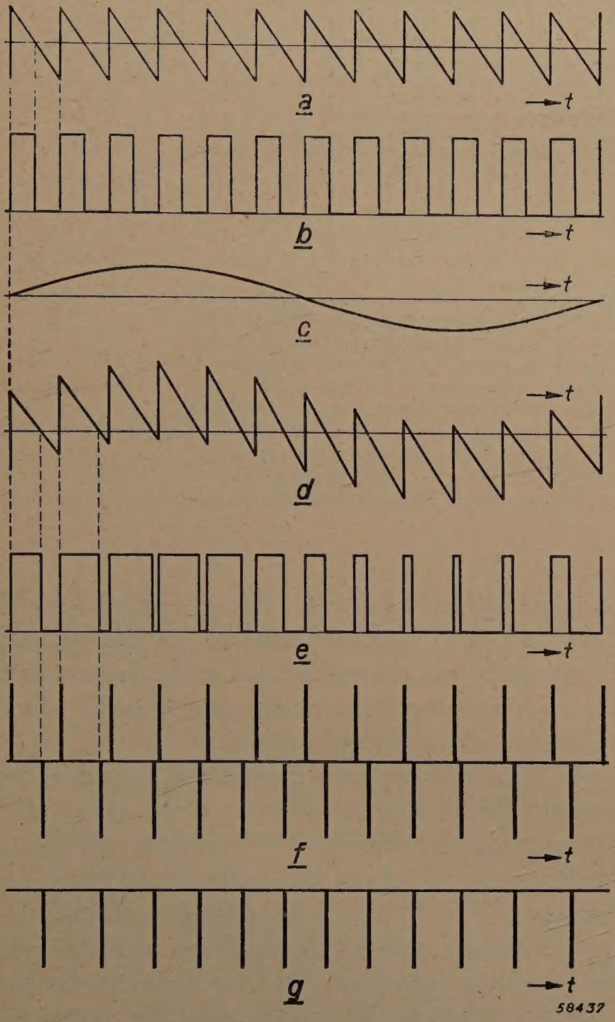

Fig. 5. Conversion of broad width-modulated pulses into narrow time-modulated pulses.
A series of identical equi-distant current pulses (*b*) is obtained by applying a saw-tooth voltage (*a*) to the control grid of a valve. If at the same time a sinusoidal voltage (*c*), i.e. a "signal", is likewise applied to the control grid then the resulting voltage assumes the shape (*d*). The pulses are then modulated in width (*e*). From this series of width-modulated current pulses one can obtain by differentiation a series of narrow positive and negative voltage pulses (*f*). When the positive pulses are suppressed, a series of time-modulated negative pulses remains (*g*).

With the aid of a differentiating network these width-modulated pulses are changed into a series of (much narrower) positive and negative pulses corresponding respectively to the beginning and the end of each of the width-modulated pulses (fig. 5*f*). Now the starting points of these latter pulses are fixed (at fixed intervals of time of 11.1 μsec), and the positive narrow pulses can therefore be cut off without any objection. What then remains are negative narrow pulses varying in time with the low-frequency modulation (fig. 5*g*).

The advantage of transmitting exclusively the narrow pulses lies mainly in the great saving of power that can thereby be obtained, since all the energy otherwise required for transmitting the broad pulses can now be used during the very short duration of the narrow pulses. With a certain average power of the transmitter the peak power in the narrow pulses can thus be made much greater.

Set-up of the modulator for eight channels

The manner in which the modulator works will now be described with reference to the block diagram given in *fig. 6*. Each rectangle in this diagram represents a group of circuit elements. Identical groups of elements are denoted by the same Roman figure (*I . . . IV*). Each of the vertical columns numbered *1* to *8* relates to one of the eight speech channels. The column marked *S* corresponds to the synchronizing channel.

Let us now consider one of the eight columns taken at random. In *I* the modulation signal (conversation) to be transmitted is amplified. In *II* the same takes place with the saw-tooth voltage, which has a frequency of 90 kc/s and is derived from the sinusoidal voltage supplied by the oscillator *A* and delivered to *B*. The superposed modulation and saw-tooth voltages are applied to the control grid of a valve represented in the diagram by a rectangle *IV*. Disregarding for a moment the rectangles *III*, in this way there would be produced in the valve *IV* width-modulated pulses having a frequency of 90 kc/s; the valve *IV* would then be alternately open and closed, on an average (in the absence of modulation even exactly) every 5.55 μsec.

However, a pulsating voltage generated by the circuit *III* (to be described farther on) is applied to a second grid (the screen grid) of the valves *IV*, which are pentodes. The result is that only one of the nine pulses is actually produced each time. These pulses then have a repetition frequency of 10 kc/s, which, as we have seen, is high enough to permit reconstruction of the modulation signal at the receiving end. As will presently be explained, the voltages supplied by the circuits *III* are such that the series of pulses derived from the columns *1 . . . 8* (fig. 6) are mutually displaced in time.

In accordance with the principle of multiplex-pulse modulation described they can therefore be placed side by side in time.

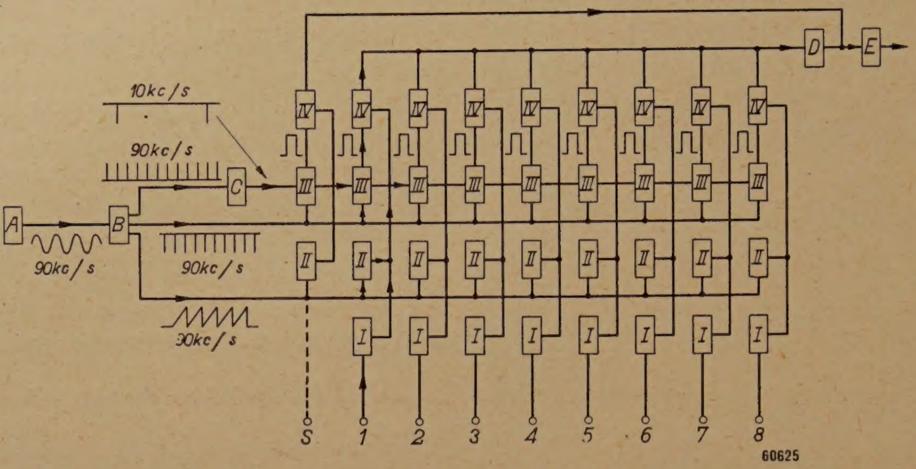

Fig. 6. Block diagram of modulator. A = an oscillator supplying a sinusoidal voltage with a frequency of 90 kc/s. In B this voltage is converted ino a saw-tooth voltage and into a series of pulses with the same frequency. In C the series of pulses with a frequency of 90 kc/s are converted into a series of pulses having a frequency of 10 kc/s. The eight columns correspond to the eight channels. Each column comprises a signal amplifier (*I*), a saw-tooth amplifier (*II*), a channel-gate oscillator (*III*) and a circuit (*IV*) in which the pulses are produced and modulated. The column S corresponds to the synchronizing signal. In D the width-modulated signal pulses are converted into time-modulated pulses. Between D and E the synchronizing pulses are added. In E the whole is amplified and limited.

The result is that the situation at the input of the differentiating circuit D corresponds to what is diagrammatically represented in *fig. 7a* (for the absence of modulation): there is a series of broad (average 5.55 μsec) pulses to be modulated in width, occurring in groups of eight (8 channels); between each two successive groups a place (11.1 μsec) is left open for the synchronizing pulse, which is not differentiated, so as to distinguish it from the others. After differentiation in D these broad and width-modulated pulses yield a series of narrow and time-modulated negative pulses and a series of narrow *un*modulated positive pulses (fig. 7*b*). To these there is added the broad synchronizing pulse obtained from column S (fig. 6), so that at the input of the circuit E (fig. 6) we get a series of pulses diagrammatically represented in fig. 7*c*. In E the whole of the input is amplified and limited (the positive pulses being cut off at the same time), after which it is fed to the transmitter.

The channel-gate circuit

We shall now describe a so-called channel-gate circuit (represented in fig. 6 by the rectangles *III*). The object of these channel-gate circuits is to ensure (1) that of the pulses which each of the valves *IV* would otherwise produce only one out of the nine actually arises, or, to express it less correctly, is passed through (hence the name channel-gate); (2) that the pulses derived from the eight channels come to lie side by side displaced in time, as represented in fig. 7*a*, and that the synchronizing pulses subsequently to be added fall in the places left open (fig. 7*c*).

As already stated, the first object is attained by making use of the screen grid of the valves *IV*; the screen-grid voltage is only positive (and constant) during one of the nine cycles of the saw-tooth voltage (90 kc/s), which is applied to the control grid together with the modulation voltage (*fig. 8a-c*); during the other eight saw-tooth cycles the screen-grid voltage is zero (fig. 8*d*). The repetition frequency of the width-modulated anode-current pulses is thus 10 kc/s (fig. 8*e*).

The second object — to ensure that the pulses of the eight channels come to lie side by side in time — is achieved by seeing to it that the positive and constant voltage (channel-gate voltage) is applied to the screen grid of a valve *IV* exactly at the right instant and during the right time, viz. 11.1 μsec, the duration of a saw-tooth cycle. In other words, each channel gate opens at the same moment as the preceding one closes, each gate remaining open 11.1 μsec.

This is done by employing a "chain" circuit, whereby the closing of a channel gate causes the

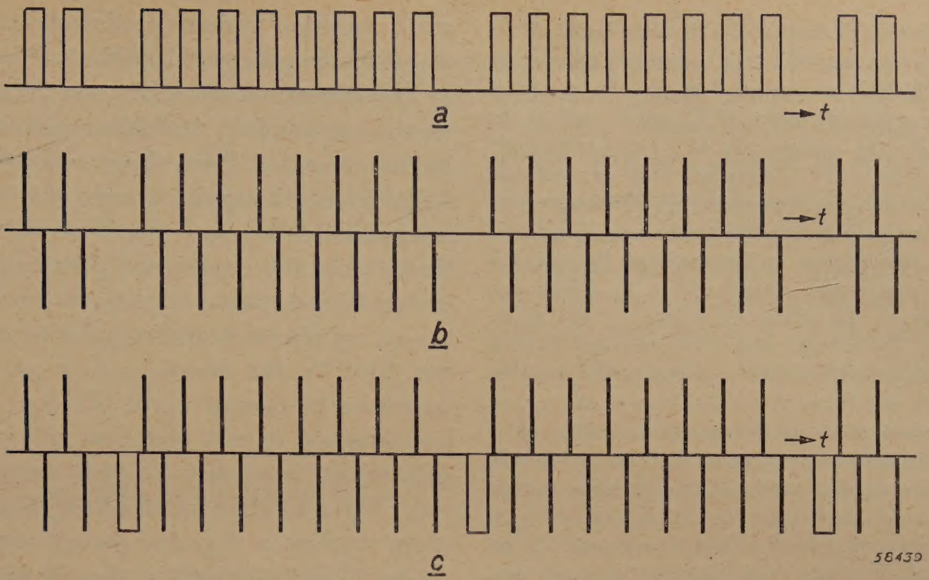

Fig. 7. *a*) The pulses corresponding to all eight channels, as they enter the differentiating circuit D (see fig. 6). For the sake of simplicity the pulses are all drawn with the same width. *b*) The same series of pulses after differentiation in D. *c*) The same series of pulses to which the series of synchronizing pulses have been added. (This corresponds to the position at the input of the circuit E in fig. 6.)

next channel gate to open. This circuit consists of nine (one for each channel and one for the synchronizing signal) identical "channel-gate oscillators" (the nine rectangles denoted by the figure *III* in fig. 6).

The diagram of such a channel-gate oscillator (*fig. 9*) resembles that of the known multi-vibrator of Abraham and Bloch. By slightly modifying the circuit, however, a certain asymmetry has been obtained so that the circuit does not oscillate continuously but has a certain position of rest to which it automatically reverts after a disturbance of the equilibrium.

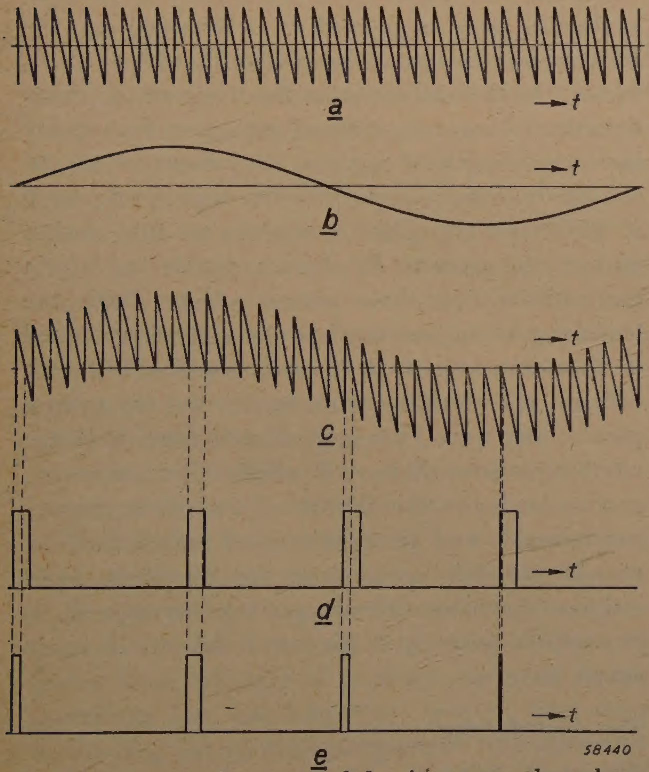

Fig. 8. Illustrating how pulses belonging to one channel are produced. The saw-tooth voltage (*a*) and the signal voltage (*b*) are applied to the control grid of a pentode (denoted by *IV* in fig. 6); superposition of these voltages produces (*c*). The channel-gate voltage (*d*) is applied to the screen grid. The resulting anode-current pulses modulated in width are represented in (*e*).

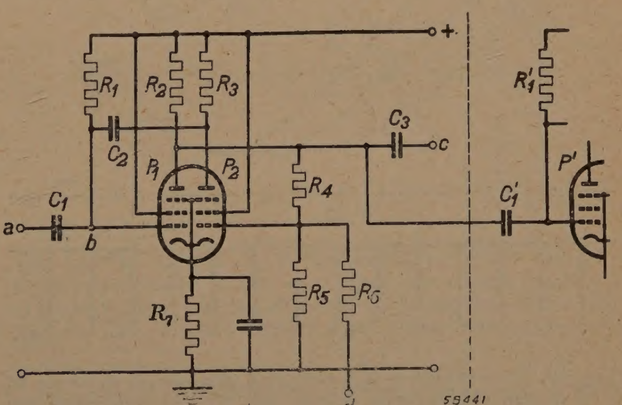

Fig. 9. Diagram of a "channel-gate oscillator". P_1, P_2 are two pentode systems in one envelope (EFF 51). The voltage at the various points (*a*, *b*, *c*, *d*) of the circuit is represented in fig. 10 as a function of time.

We have carried out this circuit with an EFF 51 valve, which contains two pentode systems in one envelope. The control grid of one pentode, P_1, is connected via a high resistance R_1 to the positive pole of the anode voltage source: thus P_1 passes current. As a result the voltage drop across the anode resistor R_2 is great and the anode of P_1 has a low voltage with respect to the cathode. The control grid of the other pentode, P_2, is connected to a tapping on the voltage divider R_4-R_5, which is connected between the anode of P_1 and earth. By a suitable choice of R_4 and R_5 and of the cathode resistor R_7, the control grid of P_2 goes so strongly negative that the pentode P_2 does not pass any current.

When at the instant t_1 the current through P_1 is interrupted by means of a negative pulse applied to the control grid via the capacitor C_1, the voltage at the anode of P_1 rises and thus also the voltage at the control grid of P_2, whereupon P_2 starts to conduct. This causes the voltage at the anode of P_2 to decrease. The voltage across the capacitor C_2 then adapts itself to the changed conditions by means of a charging current through R_1 and R_3, so that the voltage at the control grid of P_1 remains negative even in the absence of the pulse cutting off P_1. This new situation, where P_1 is not conducting and P_2 is conducting, will last until at t_2 the charging current of C_2 has become so small that the voltage across R_1 is not sufficient to keep P_1 cut off. Then P_1 will again begin to pass current and ultimately the initial situation is restored.

Fig. *10a-c* represents the voltage at various points of the circuit as a function of time. From fig. 10*c* it can be seen that the voltage at the anode of P_1 (fig. 9), which voltage is taken off via the capacitor C_3, has the shape required for a "channel gate" for the screen-grid voltage of a pentode *IV*, provided the time t_2 to t_1 is equal to 11.1 μsec and the negative pulse is repeated with a frequency of 10,000 c/s.

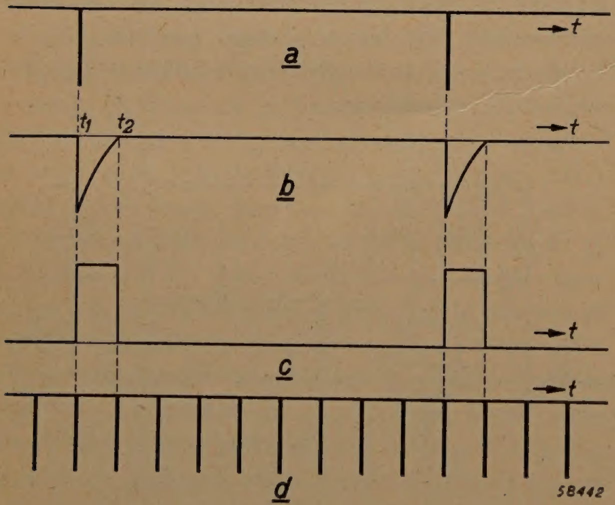

Fig. 10. The voltage as a function of time at the points *a*, *b*, *c*, *d* of the channel-gate circuit (fig. 9). (*a*) represents the pulse activating the circuit, (*b*) the voltage at the control grid of P_1, (*c*) the voltage at the anode of P_1 (the channel-gate voltage), (*d*) the auxiliary pulses fixing the duration of the channel-gate voltages at 11.1 μ sec.

The manner in which nine such circuits are interlinked and bring about the opening and closing of the nine channel gates is as follows. The first circuit — rectangle *III* in column *S* of the block diagram given in fig. 6 — is activated with negative pulses produced in *C* with the repetition frequency of 10,000 c/s. For the activation of the second circuit, 11.1 μsec later, the voltage at the anode of P_1 is "differentiated", so that at the moment that this voltage returns to its original value a negative pulse occurs; this negative pulse is applied to the control grid of P_1' of the second circuit (via the capacitor C_1' corresponding to C_1 of the first circuit). In the same way the second circuit supplies a negative pulse for the third one, the third for the fourth, and so on. This "chain" continues to operate until the last (ninth) circuit has returned to rest and thus the whole channel-gate circuit is at rest. At that moment, 100 μsec after the activation of the first circuit, the latter again receives a negative pulse from *C*.

The "differentiation" of the anode voltage of P_1 is brought about by the elements C_1', R_1', P_1' of the following channel-gate oscillator. Owing to the control-grid current passing through R_1' the control-grid-cathode part of P_1' has a low internal resistance forming together with the capacitor C_1' the differentiating circuit.

It is highly important that all the channel gates remain open exactly the same length of time, namely during exactly one cycle of the saw-tooth voltage, i.e. 11.1 μsec. Now the time during which a channel gate is opened is determined by the *RC*-time of the circuit formed by C_2 and $R_1 + R_3$ (fig. 9). In this circuit there lies a source of errors. In order to keep the channel gate open for exactly the same length of time as the duration of the saw-tooth cycle, a negative pulse with a frequency of 90,000 c/s (fig. 10*d*) is applied to the control grid of the pentode P_2 of each double valve (via the resistor R_6); these pulses coincide with the beginning of the saw-tooth cycles and are produced in the same circuit *B* as produces the saw-tooth voltage (see block diagram, fig. 6). For the greater part of the time P_2 is cut off and these negative auxiliary pulses will have no effect. When, however, current begins to flow through P_2 and if the channel gate should tend to remain open longer than the prescribed 11.1 μsec, then the auxiliary pulse ensures that the current passing through P_2 is interrupted after 11.1 μsec and the circuit again comes to rest.

The demodulator

The anode follower

Let us now see what happens at the receiving end. The high-frequency wave trains are picked up by an aerial in the normal way and led to the high-frequency part of a receiver. After detection this part produces pulses of the same form as those supplied to the transmitter. Now these pulses first have to be separated into series corresponding to the individual low-frequency modulation signals (conversations). Then each series has to be demodulated, that is to say the low-frequency modulation signal has to be reconstructed from each series. We shall now explain how these two stages work in our apparatus, though not in the same order in which they have just been mentioned.

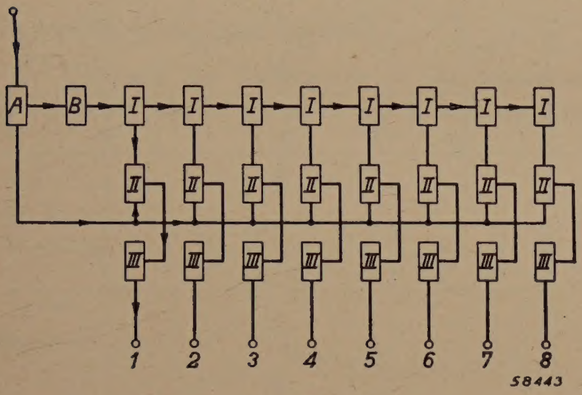

Fig. 11. Block diagram of the demodulator. A = amplifier and limiter; B = circuit in which the synchronizing pulses are separated from the other pulses; I = channel-gate oscillators; II = "anode followers"; III = amplifiers and low-pass filters. The eight columns correspond to the eight channels.

Fig. 11 is a block diagram of the demodulator. Each of the eight columns numbered *1* to *8* corresponds to one channel (conversation). The rectangle *I*, the channel-gate circuit, will be dealt with later. In *II* the channels are separated with the aid of the channel-gate voltages supplied by *I*, whilst at the same time the low-frequency speech frequencies are reconstructed from the separated pulses belonging to one channel. As regards the circuit *III* there is nothing much to be said. It contains a low-frequency amplifier and a low-pass filter, the latter "smoothing out" the signals having the speech frequencies. The output is matched to the telephone line connected to it.

The circuit *II*, however, calls for a more detailed explanation. Once the pulses belonging to one channel have been separated from the others (the manner in which this takes place in *II* will be shown later on), then in principle we could obtain therefrom the low-frequency speech component by filtering out all other frequencies, the repetition frequency and its harmonics, with the aid of a low-pass filter. Since, however, the desired component is only weakly represented in the frequency spectrum of the pulses, such a method requires a great deal of amplification and very selective filters. The same applies for other methods whereby time-modulated pulses are reconstructed into width-modulated pulses or converted into amplitude-modulated pulses. In our installation, however, an entirely different method of demodulation is followed, namely with the aid of a circuit whereby use is made of a secondary-emission valve and which belongs to the group of "anode-follower" circuits (so-called because, with a suitable choice of components, the potential of the auxiliary cathode follows the anode voltage variations).

To explain the action of an anode follower we must consider the characteristics of a secondary emission valve which represent the relation between the current I_{k2} and the voltage V_{k2} of the auxiliary cathode k_2, with the anode voltage V_a as parameter and with a constant voltage on the control and screen grids (*fig. 12*).

When the value of V_{k2} is very small only a few electrons emerge through the openings in the screen grid and reach k_2; in other words, there is a weak current I_{k2}. The direction of this current (electrons striking k_2) will be called positive.

As V_{k2} increases (but for the time being still remaining smaller than V_a) the electrons impinging upon k_2 set up secondary emission, the number of electrons leaving k_2 per second exceeding the number reaching it, so that I_{k2} is reversed. The secondary electrons move towards the anode, which has a higher potential than k_2.

When V_{k2} is allowed to exceed V_a the secondary electrons are unable to reach the anode and return to k_2, so that I_{k2} again becomes positive; actually k_2 then plays the part of an anode. When $V_{k2} = V_a$, $I_{k2} \approx 0$.

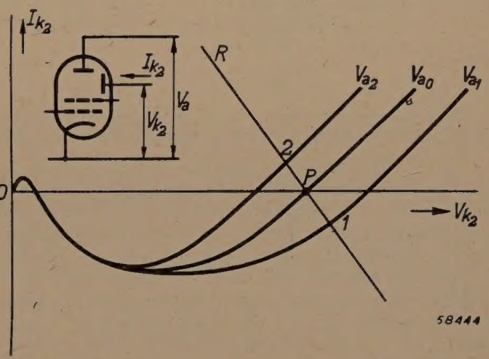

Fig. 12. The current I_{k2} in the auxiliary-cathode lead of a secondary-emission valve as a function of the auxiliary-cathode voltage V_{k2} for different values of the anode voltage V_a. The oblique straight line is the "resistance line".

If a resistor R is connected in series with k_2, and using V_b to denote the supply voltage of the auxiliary cathode circuit, then upon the value of V_a being varied I_{k2} becomes equal to zero when $V_a \approx V_b$. For any other value of V_a (say V_{a1} or V_{a2}) I_{k2} adjusts itself to the point of intersection (*1* or *2* respectively) of the respective characteristic on the "load line", i.e. the straight line drawn with a slope = $1/R$ through the point P on the abscissa with $V_{k2} = V_b$. If R is very large then this line is almost horizontal and the points *1*, *2* fall practically on the abscissa, that is to say V_{k2} is then always approximately equal to V_a. Hence the name "anode follower".

Demodulation of a channel with the aid of the anode follower

The manner in which the anode follower is used for demodulating the pulses is as follows. The circuit [2]) is represented in *fig. 13.* The control grid, the screen grid and the anode of a secondary-emission valve EFP 60 serve as input electrodes, and the auxiliary cathode as output electrode. The whole group of pulses of all the channels is applied to the control grid, a channel-gate voltage (supplied by the circuit *I*, fig. 11) is applied to the screen grid, whilst there is fed to the anode — in addition to a direct voltage — a saw-tooth voltage derived via an integrating network from the channel-gate voltage. The auxiliary cathode is fed via a resistor *R* (shunted by a capacitor *C*) from the same voltage source as the anode and is connected to the output terminal via a blocking capacitor. The channel-gate voltage ensures that the valve is open only for the respective channel and cut off for all other channels. (Where in the further description of this apparatus we speak of pulses only the pulses of one channel are meant.)

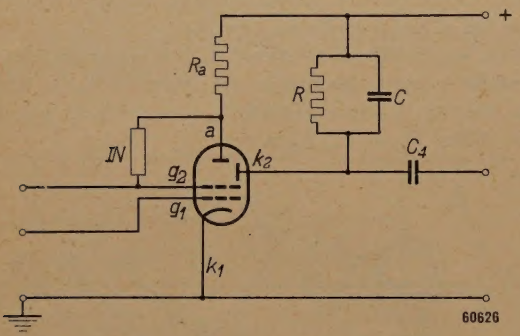

Fig. 13. Anode follower with secondary emission valve (EFP 60) for demodulating one channel. Input electrodes: control grid g_1, screen grid g_2 and anode a. Output electrode: auxiliary cathode k_2. The main cathode is denoted by k_1. *IN* = integrating network. *R-C* = circuit with a relaxation time which is long compared with the interval between two successive pulses. C_4 = blocking capacitor. R_a = anode resistor.

In accordance with the property of the anode follower, with each pulse the auxiliary cathode adjusts itself to the instantaneous value of the anode voltage. If there is no modulation then the pulses are in the intermediate position, whereby the anode voltage has just reached half way up the steeply ascending side of the saw-tooth. When modulation is present the pulses move to the left or to the right, with corresponding values of anode voltage lower or higher, as the case may be, in proportion to the displacement of the pulses. The auxiliary cathode follows these fluctuations in the anode voltage, so that the valve acts, as it were, as a switch momentarily connecting the auxiliary cathode to the anode at every pulse. When the pulse has passed, the auxiliary cathode voltage is maintained because the relaxation time *RC* is long compared with the interval of time between the pulses. This is made clear in *fig. 14a-d,* where the bottom curve represents the step-like variation of the output voltage. This voltage is further amplified in *III* (fig. 11).

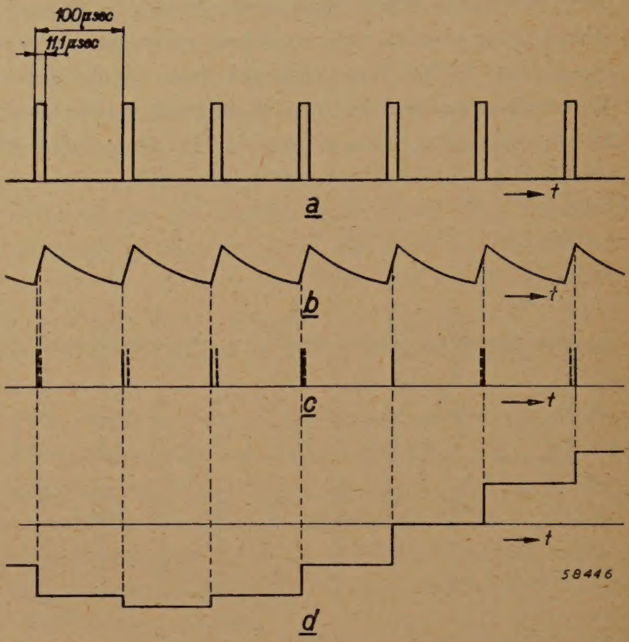

Fig. 14. The voltage as a function of time at various points of the circuit indicated in fig. 13: (*a*) channel-gate voltage at the screen grid, (*b*) saw-tooth voltage at the anode obtained from (*a*) by integration, (*c*) time-modulated signal pulses at the control grid (state of rest indicated by dotted lines), (*d*) output voltage (auxiliary-cathode voltage with the D.C. component removed).

As already observed, in the case of pulse modulation only a few instantaneous values of the low-frequency voltage are transmitted (*fig. 15a*).

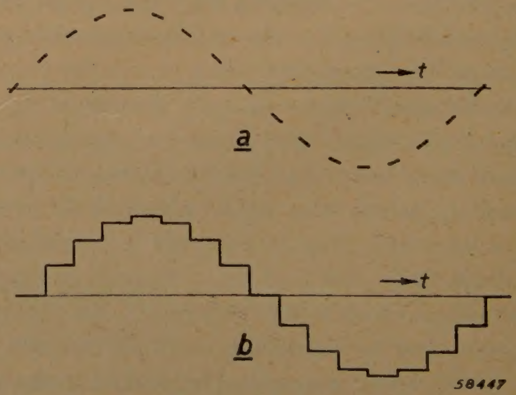

Fig. 15. Of a signal having the shape of a sinusoidal curve only a sinusoidal succession of dashes (see fig. 3) is transmitted such as represented in (*a*). By demodulation a step-like curve (*b*) is obtained therefrom.

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[2]) For this circuit we are indebted to Tj. Douma (Philips Research Laboratory).

Since it is not known whether the pulse next in succession corresponds to a larger or a smaller amplitude, the step-like curve illustrated in fig. 15*b* is the best approximation of the signal that can be expected from the curve of successive dashes (fig. 15*a*). The step-like curve, contrary to the pulses, contains the desired low-frequency signal to a high degree. A simple low-pass filter suffices to filter out the undesired higher harmonics, so that there is no need for very selective filters or for any great amplification.

Separation of the eight channels

To complete the description of the functioning of the demodulator we must say something about the manner in which the channel-gate voltages applied to the screen grid of the secondary-emission valves are derived. These voltages are represented in fig. 14*a* as a function of time. Since the installation has been designed for eight channels, there are eight such voltages required, so displaced in time that at any moment only one of them is positive. The same problem has already been encountered in the description of the modulator, and the solution provided there in the channel-gate circuit has likewise been applied for the demodulator. The eight rectangles *I* in the block diagram of fig. 11 represent in fact the same channel-gate circuit as the rectangles *III* in the block diagram of fig. 6 (in the demodulator there is no rectangle corresponding to the synchronizing signal as given for the modulator).

From the description of the channel-gate circuit we have seen that this circuit has to be repeatedly activated with a negative pulse, having a repetition frequency of 10 kc/s. For the demodulator the series of negative pulses required for this purpose is derived from the series of broad synchronization pulses, likewise having the repetition frequency of 10 kc/s [3]). This is done with the aid of a circuit denoted in the block diagram of fig. 11 by the letter *B*. All pulses, including the synchronizing pulses, are fed to this circuit and "differentiated". This differentiation is brought about with the aid of an *RC*-circuit so dimensioned that the *RC*-time is long compared with the duration of the signal pulses but short with respect to the duration of the synchronizing pulses (which are of course much wider). The result is that after the differentiation there is a positive voltage peak corresponding to the end of each synchronizing pulse which is much higher than the positive peaks corresponding to the end of each signal pulse (*fig. 16*) [4]). This makes it possible to separate the voltage peaks belonging to the synchronizing pulses from the other peaks with the aid of a valve, which then supplies the requisite negative pulses. This valve also forms part of the circuit *B*.

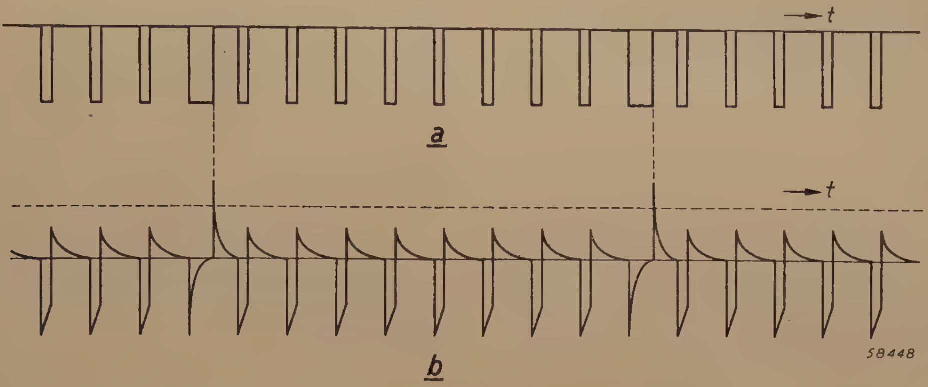

Fig. 16. *a*) The pulses corresponding to all eight channels and the synchronizing pulses, in the form in which they reach the input of the circuit *B* (fig. 11).
b) The same pulses after differentiation with a differentiating network forming part of the circuit *B*.

Signalling

As is known, signalling — the transmitting of the dialling note, the dialling pulses, the "engaged" signal, etc, — is an important aspect of telephone traffic. The technical problem of signalling, which in carrier telephony is rather difficult [5]), is very easily solved with the form of multiplex-pulse modulation described here. With this method signalling is done by the transmission of direct-current pulses, just as is the case with local telephone traffic. As seen from the description of the demodulator, a fixed displacement of a pulse results in a certain change in the direct voltage at the auxiliary cathode of the anode follower. Consequently the D.C. dialling pulses, displacing a certain signal pulse from the state of rest, will cause analogous direct-voltage pulses at the auxiliary cathode, which can be amplified with a D.C. amplifier and then fed to the respective relays.

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[3]) It is to be noted that in the demodulator there are no auxiliary pulses with the repetition frequency of 90 kc/s which in the modulator close the channel gates at the right moments. Here the time during which the channel gates are open thus depends entirely upon the *RC*-time of the circuit formed by C_2 and $R_1 + R_3$ (fig. 9). The right adjustment of this time has to be obtained by adjusting, for instance, the capacitor C_2.

[4]) See: Projection-television receiver, V. Synchronization, by J. Haantjes and F. Kerhhof, Philips Techn. Rev. **10**, 364-370, 1949 (No. 12).

Final remarks

It remains to say something about the field of application of the apparatus described here for multiplex-pulse modulation.

Obviously this method of modulation is not likely to be applied in the case of beam transmission that has to form a link in a carrier-telephony system, for this would involve a double apparatus both at the transmitting and at the receiving end, namely at the transmitting end an apparatus for carrier-wave demodulation and one for pulse modulation, and at the receiving end converse apparatus. In such a case therefore a method of transmission will be sought whereby the carrier is transmitted in its entirety [6]). (In general one must then be satisfied with longer and thus less easily beamed waves.) If, on the other hand, there are not too many "detached" conversations to be put through (reaching the transmitter in a form other than that of a carrier-wave band) then the apparatus described here for multiplex-pulse modulation has great advantages, in that it is comparatively simple and inexpensive.

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[5]) See for instance: F. A. de Groot, Signalling in carrier telephony, Phiilps Techn. Rev. 8, 168-176. 1946.

[6]) A. van Weel, An experimental transmitter for ultra-short-wave radio-telephony with frequency modulation, Philips Techn. Rev. 8, 121-128 and 193-198, 1946.

Summary. In beam-transmission links difficulties arise which can be met by the application of pulse modulation. In this article a brief explanation is first given of the principles of pulse modulation and of the so-called multiplex-pulse modulation, i.e. the transmission of several telephone channels with the aid of pulse modulation. Next a description is given of a new apparatus for multiplex-pulse modulation with eight channels. In the modulator the succession of pulses belonging to the various channels are obtained by means of a circuit consisting of a chain of channel-gate oscillators. In the demodulator the various channels are separated by a similar chain circuit. Demodulation itself is done with the aid of a circuit employing a special property of the secondary-emission valve (anode follower).

PIEZO-ELECTRIC MATERIALS

by J. C. B. MISSEL.

537.228.1

The intention is to publish in this periodical some articles giving an impression of the great advance made in the manufacture of quartz-oscillator plates in the U.S.A. during the last war and of the part played therein by the North American Philips Company. As an introduction to these articles, and to others which may be published dealing with more recent developments in this field, it is deemed advisable first to review some of the most important facts and conceptions of piezo-electricity, with particular attention to new piezo-electric materials.

The literature about the theory and practice of the piezo-electric effect has already reached a respectable size. This phenomenon and its application have numerous aspects deserving of consideration, and for the initiated the few pages that will be devoted to the subject here cannot but give the impression of being far from complete. For this the author is to be excused on the ground that this introduction is only intended to make a number of articles to be published on the subject in this periodical understandable without the reader having to consult other literature. For those who wish to go into the subject more deeply some extensive publications are quoted at the end of this article.

Piezo-electricity and its application

The phenomenon of piezo-electricity was discovered by Pierre and Jacques Curie in 1880. They found that under a mechanical load a quartz crystal exhibits electrical charges of opposed polarity on two opposite faces (that is to say it becomes an electrical dipole). Crystals possessing this property always show also the converse effect: when placed in an electric field they exhibit not only electrical polarization but also mechanical deformation. In both cases the effect is linear, that is to say the charges and the deformations are proportional to the causative forces and electrical field strength, and reverse in polarity together with them. Thus the inverse piezo-electric effect can be immediately distinguished from the phenomenon of electrostriction, i.e. the deformation that any substance, crystalline or amorphous, undergoes in an electric field and which varies with the square of the field strength.

The strength of the piezo-electric effect depends mainly upon the material and the direction in which the mechanical load (e.g. a pressure) is applied. The effect is very strong in the case of Seignette salt (Rochelle salt, potassium sodium tartrate): in a certain direction of the crystal a charge can be obtained amounting to about 8×10^{-9} coulomb per newton of the force applied (2.3×10^{-4} e.s.u./dyne) ¹). With quartz one gets in the most favourable direction about 2.10^{-12} C/N.

The practical applications that have been found for the piezo-electric effect are so commonly known that they need only be referred to briefly here. Its most obvious use is for indicating mechanical vibrations by electrical means and, conversely, for generating mechanical vibrations by means of alternating voltages. The first possibility has been realized in the crystal pick-up and the crystal microphone, and the second possibility in supersonic radiators. The powerful supersonic waves emitted by these radiators find practical application for signalling under water, for preparing very finely distributed emulsions and suspensions, and also for tracing defects in large, massive pieces of material. In an entirely different way piezo-electric crystals are used as elements for stabilizing the frequency of radio transmitters, for which purpose during the last war quartz oscillator plates were turned out in tens of millions in the U.S.A. and elsewhere. Finally, there is to be mentioned the use of piezo-crystals as filter elements (transducers) for carrier-telephony and other branches of communication engineering.

The reasons why piezo-crystals are suitable for these purposes will be mentioned below.

Piezo-electric species of crystals

With one single exception piezo-electric effect may be expected in all species of crystals not

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¹) Or, in the case of the inverse effect, a deformation of 8×10^{-9} metre per volt applied. From the theory it follows that the proportionality coefficients for the direct and the converse effect are identical; it is easily verified that, as regards the dimensions, there is nothing contradictory in this (C/N is the same dimension as m/V). Thus the substance having the greatest direct effect has also the greatest converse effect.

²) See, e.g., Philips Techn. Rev. 5, 145, 1940, and 5, 9, 1940 (laryngophone).

exhibiting a centre of symmetry in their crystal lattice. The fact that this effect cannot occur in crystals having a centre of symmetry is easiest understood when examining an elementary cell in the lattice of such a crystal (see *fig. 1*). What the piezo-electric effect amounts to is the accompanying of a deformation of a crystal (thus also of the elementary cell) by a mutual displacement of the centres of gravity of the negative and positive charges carried by the ions in the cell. Let us draw from the centre of the elementary cell a radius vector to one of the ions. If the crystal lattice has a centre of symmetry the opposite and equally long radius vector will end in an ion of the same kind; the centres of gravity referred to therefore both lie in the centre of the cell. For the very reason of the symmetry existing also in the mechanical strength of the crystal in various directions, in the case of a deformation due to external forces the property of the centre of symmetry is maintained; thus the centres of gravity of the positive and the negative charges continue to coincide and there is no piezo-electric effect.

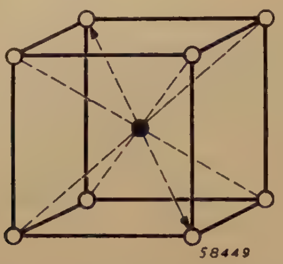

Fig. 1. Simple example of a crystal species with centre of symmetry; elementary cell of the crystal lattice of caesium chloride, CsCl.

The complete lattice is formed by displacing the cell in the direction of the ribs of the cube one rib-length at a time. The white ion may be caesium and the black one chlorine, or vice versa. The lattice is of course electrically neutral, although the elementary cell apparently contains eight white ions and only one black one: each of the white ions belongs simultaneously to eight elementary cells bordering on each other in one corner and may, therefore, be counted as belonging for only one-eight part to the cell drawn. Similar considerations apply for the types of crystals represented in the other illustrations to follow.

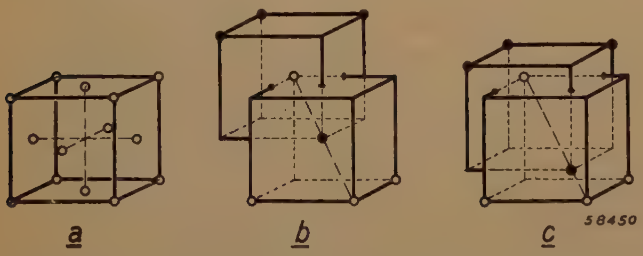

Fig. 2. *a*) Face-centered cubic lattice of one kind of ion.

b) The crystal lattice of the rock-salt type can be constructed by interlacing two lattices according to (*a*), one of "white" ions (e.g. sodium), the other of "black" ions (e.g. chlorine), in such a way that one is displaced, with respect to the other, over half the length of the space diagonal of the elementary cell.

c) The crystal lattice of the sphalerite type is formed similarly to (*b*), but with the "white" lattice (e.g. copper) and the "black" one (e.g. chlorine) displaced with respect to each other over only a quarter of the length of the said space diagonal.

In (*b*) and (*c*) only a few ions of each lattice are shown, for the sake of clarity.

After this somewhat abstract reasoning the reader may feel the need of an example. For this we shall choose alkali-halogenides, which do not exhibit any piezo-electric effect, and cupro-halogenides, which do show that effect. The crystallization of these substances is of the rock-salt type and the sphalerite type respectively. Both crystals contain only two kinds of ions and can be built up from two face-centered cubic lattices of the two kinds of ions interlaced in a certain way. An attempt has been made to illustrate this in *fig. 2*. From the elementary cells drawn in *figs 3a* and *4a* it can be verified that the rock-salt type possesses a centre of symmetry whereas the sphalerite type does not.

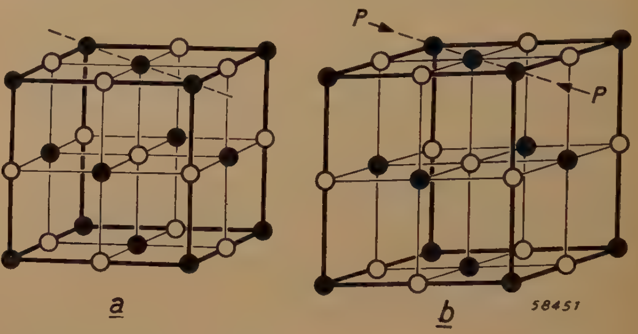

Fig. 3. Elementary cell of a crystal of the rock-salt type (*a*). Through deformation by the indicated forces *P* the cell assumes the shape (*b*): the originally square top face becomes a rhomboid. In the original as well as in the deformed state the cell has a centre of symmetry in which lie the centre of gravity of all white ions as well as that of all black ions. Thus there is no piezo-electric effect.

Can it be understood how the piezo-electric effect is brought about in the sphalerite type? In the normal state (fig. 4*a*) also with this type the centres of gravity of the "white" and the "black" ions both lie in the centre of the elementary cell. Upon the elementary cell being deformed by a pressure acting in the direction indicated (*P*) all the ions move towards each other in the direction of the arrow and away from each other in the directions perpendicular thereto (fig. 4*b*). Taking into account the fact that the binding forces of all neighbouring ions keep each ion in its position more or less resiliently, it will be clear that as a result of the black ions *A* and *B* approaching each other and the black ions *C* and *D* moving away from each other the white ion *E* will fall deeper between the ions *C* and *D*. This applies similarly for the other three white ions *F*, *G* and *H* in the elementary cell. In the deformed elementary cell (fig. 4*b*) the centre of gravity of the white ions will therefore have dropped with regard to that of the black ions.

Here one can see at once how the piezo-electric effect arises by virtue of the absence of a centre of symmetry: if there were also white ions at the points *I*, *K*, *L*, *M* then the same deformation would cause these to be forced upward and the centre of gravity of all white ions would not be displaced.

Such a configuration does actually occur: it is the so-called fluorite type in which, for instance, calcium fluoride crystallizes; see *fig. 5*. As is to be expected, this kind of crystal, which again has a centre of symmetry, is not piezo-electric.

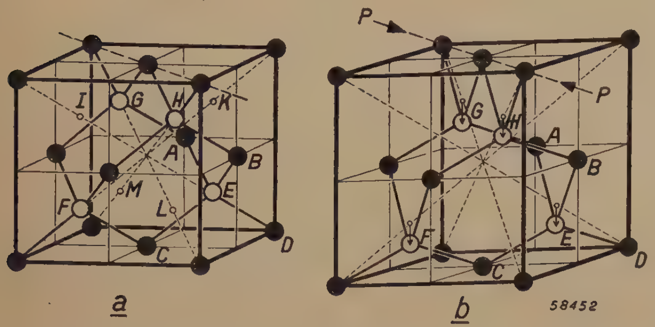

Fig. 4. Elementary cell of a crystal of the sphalerite type (*a*). There is no centre of symmetry. In the case of a deformation similar to that in fig. 3 the cell assumes the shape of (*b*). The white ion *E* has then dropped with respect to the four surrounding black ions *A*, *B*, *C*, *D*. The same applies for the other three white ions *F*, *G*, *H*, so that the centre of gravity of the white ions in the cell now lies lower than that of the black ions (piezo-electric effect).

About 1000 of the 10,000 species of crystals so far identified crystallographically belong to the classes without centre of symmetry and thus exhibit a piezo-electric effect. Unfortunately, there are only a few of these that can be considered for practical use, since in the first place one must have rather large crystals. Of the natural species of crystals only quartz, tourmaline and zinc blende can be considered. Tourmaline crystals of a sufficiently homogeneous structure very seldom occur. Zinc blende is difficult to work with because it readily splits. Thus we are left with quartz as the only useful — but very useful — natural piezo-crystal. Of the artificial crystals having piezo effect the Seignette salt already mentioned has been known the longest. Some other synthetic piezo-crystals will be mentioned below.

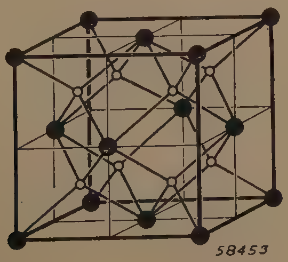

Fig. 5. When the places *I*, *K*, *L*, *M* in fig. 4*a* also become occupied by "white" ions a crystal lattice with a centre of symmetry is again formed, the fluorite type (example: black ions calcium, white ions fluorine; CaF_2). When deformation takes place four white ions drop and four white ions rise, the centre of gravity remaining where it was — hence no piezo-electric effect.

Properties of Seignette salt and quartz as piezo-electric materials

It has been stated above as a practical requirement that one must have piezo-crystals of a sufficiently large size. Of course there are also other criteria of the usefulness of a piezo-crystal. The foremost of these are: the strength of the piezo effect; the mechanical strength, determining what deformations can be allowed; the susceptibility of these properties for such external conditions as temperature, humidity, etc.

The piezo-electric effect of Seignette salt is exceptionally great, but the crystals of this salt can only be used between —30 and 50 °C, they are soluble in water and they have little mechanical strength. Owing to this last-mentioned characteristic the use of Seignette salt is confined mainly to microphones and pick-ups; for such apparatus a forced vibration with greatly divergent frequencies is essential. Consequently the crystal cannot be allowed to act in the vicinity of one of its mechanical resonant frequencies and one has always to do with relatively small deformations which the Seignette salt crystal can still well withstand. It is for the very reason of the smallness of the deformations that the extremely great piezo effect of Seignette salt is essential. As regards its susceptibility to moisture and temperature, in the applications mentioned the conditions can be sufficiently stabilized, so that there are no objections on this account to this use of Seignette salt.

Seignette salt is used also as a generator for mechanical, supersonic vibrations, though as a rule quartz is more suitable for this, especially where it is a matter of large powers. The piezo-electric effect of quartz is, it is true, by no means so strong as that of Seignette salt, but with quartz the permissible deformations are much greater and in this case one can easily get large deformations since in principle only one frequency is to be generated at a time and it can be arranged for this to be a natural frequency of the crystal plate. Waves can thus be produced having a power of about 10 W per cm² of the radiating surface, which is much more than can be reached with electrodynamic loudspeaker systems or by means of magneto-striction, which is also sometimes applied.

Quartz likewise possesses excellent properties for other applications of the piezo-electric effect, in particular for the above-mentioned frequency-stabilization of radio transmitters. This we shall explain with reference to the electrical equivalent circuit of a vibrating piezo-electric crystal given in *fig. 6*. The self-inductance L represents the vibrating mass, the capacitance C_1 the mechanical rigidity and C_2 the static capacitance determining the voltages arising in the crystal under the varying charge due to the periodical deformation. The resistance r represents the various causes of damping: internal friction of the crystal, damping due to the supports and the surrounding electrodes, and damping due to possible radiation of acoustic waves. The quantities depend upon the species of crystal, the shape and crystallographic orientation of the crystal plate, and the conditions under which it is used. For quartz one can reckon in practice on values of say $L = 10$ henries, $C_1 = 0.1$ μμF, $C_2 = 20$ μμF, $r = 200$ ohms.

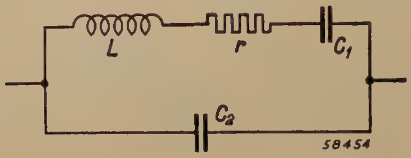

Fig. 6. Electrical equivalent circuit of a piezo crystal vibrating under the influence of an alternating voltage applied to it.

When the crystal is used in the oscillator circuit of a transmitter this circuit can as a rule only oscillate in the frequency range in which the reactance of the crystal is positive. We shall not here write down the formulae for the impedance $R + jX$ of the equivalent circuit (fig. 6) but merely give a diagram of the reactance X as a function of the frequency; see *fig. 7*. This diagram shows a series resonance ($X = 0$) at a frequency f_s and a parallel resonance ($X = \infty$ or at least very large) at a frequency f_p. Between these frequencies $X > 0$ and the circuit is able to oscillate. If C_2 is much greater than C_1, as is the case with quartz, then f_p and f_s lie very close together (in the case of quartz the difference between f_p and f_s is only a few tenths of a percent of f_s) and the frequency at which the circuit oscillates depends almost entirely on the values of L and C_1.

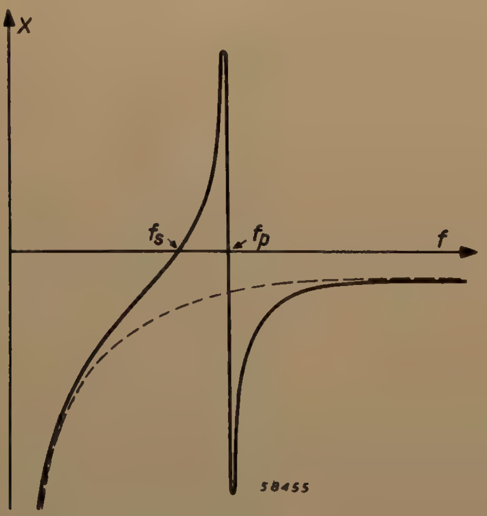

Fig. 7. The vibrating piezo crystal has an impedance $R + jX$. Here the reactance X is plotted as a function of the frequency f. If C_2 is much greater than C_1 then in the greater part of the frequency range the crystal behaves as a capacitance, $X = -1/\omega C_2$ (see the dotted hyperbola). At the frequency f_s a series resonance arises, the impedance becomes real and has the value $R \approx r$ (see fig. 6). At f_p there is a parallel resonance, X becoming very large (in the case $r = 0$, infinite).

When the above-mentioned values of L and C_1 are compared with those of a circuit made from coils and capacitors it is noticed that the desired tuning with a quartz crystal is reached at a high value of L and a small value of C_1. Since r is also fairly small, this means that the quality factor $Q = (1/r)\ \sqrt{L/C_1}$ of the crystal is much higher than that of an LC-circuit. At frequencies of 10 Mc/s normal LC-circuits, which are still sometimes used, have at their best a quality factor $Q \approx 300$, whereas with quartz crystals one can reach a value of about 30,000, and even 200,000 in vacuo (there then being no damping from acoustic radiation). Thanks to this high quality factor the oscillation frequency is very well stabilized.

It is further of great practical importance that the values of L and C, or rather the resonant frequencies, of quartz are highly stable. These can in particular be made highly independent of temperature by cutting the oscillator plate out of a crystal in certain crystallographic directions. In the orientations most used, the so-called AT and BT cuts, the relative change in frequency of the oscillator plate over a range of 100 degrees Centigrade is no more than some hundredths percent, and with the so-called GT cut even no more than 0.0001%. By way of illustration it may be mentioned that with such a crystal placed in a thermostat and forming part of an oscillator circuit having little effect upon the crystal frequency, a clock can be built which measures the time with an error of no more than 1 in 10^9. This is less than the error arising in astronomical time recording due to the irregularities in the rotation of the earth.

This great degree of constancy is also due in part to the fact that quartz is very hard and highly insusceptible to chemical action. In fact, a tolerance of 10^{-5} in the natural frequency of an oscillator plate would already be exceeded if a minute layer some millionths of a millimetre thick were to be chipped off the surface of the plate. Thanks again to its great hardness a quartz plate can be processed with extreme accuracy, so that not only can the highly constant frequency be obtained but it can also be predetermined with a tolerance of say 10^{-5}.

After having summed up all the good properties of quartz, we must point out the restrictions to its use. We have already mentioned the case of microphones and other apparatus not operating with resonant vibrations, for which the piezo effect of quartz is not great enough. To this we may add the application of the piezo effect for transducers in carrier-telephony. Such band-pass filters must have a transmitting range of the width of a speech channel (bandwith about 3400 c/s). This width can be reached with a smaller carrier frequency and/or smaller number of filter elements (crystals) according as the percentage difference between f_p and f_s (see above) of the crystal species is greater. As we have seen, in the case of quartz this difference is small, so that, particularly with low carrier frequencies of, say, 20 kc/s, quartz cannot be used directly.

New piezo-electric materials

The cases mentioned in which quartz is not suitable have led to the development of new piezo-electric materials, a few of which will be referred to below. But also for those cases where quartz would be excellently suitable one cannot, unfortunately, say that the problem of the piezo-electric material has been solved; nature's store of quartz crystals on this earth is not inexhaustive. It is already to be foreseen that the production from the known sources of quartz (mainly Brazil and less so Russia, Japan, Madagascar, and Spitzbergen) will in course of time prove to be too small to meet the demand.

To defer the moment when such an impasse will be reached, means are being sought for making at least the most economical use of the limited stock available, for in this direction there is still much to be desired. At the moment the position is such that the oscillator plates produced represent only 10% by weight of the rough crystals from which they are made, the other 90% being lost mainly through two causes: 1) about 40% is pulverized in the sawing of the crystal into plates, which loss cannot very well be avoided; 2) about 50% of the material does not reach even the sawing stage, because more or less large pieces of practically every natural crystal show all kinds of faults, such as impurities, air inclusions, and particularly the phenomenon of twinning. Where twinning occurs the crystallographic axes in different parts of the crystal are differently orientated, and in an oscillator plate this would result in different parts of the plate reacting to a voltage with different deformations, so that nothing would come of the desired oscillation. Means have now been found ³), and already applied with some success, to "detwin" a natural crystal, so that a much larger portion of the rough material can be utilized.

Of more importance for the future are the attempts that have been made to produce quartz crystals artificially. It has been found possible to produce mono-crystals of quartz with linear dimensions of 2 or 3 cm in the space of a few weeks' time in autoclaves under a pressure of, for instance, 1000 atmospheres. Similar results have been attained also with lower pressures ⁴). It is, however, questionable whether such methods can be developed for large-scale manufacture of a product at a sufficiently low price.

Another course of development is the attempt to find new piezo-electric materials not occurring in nature and which can be synthetized in the form of sufficiently large crystals to be used as substitute for quartz. This brings us to the already-mentioned development of synthetic crystals which are not intended to replace quartz but to supplement it in certain fields of application.

In the search for these new piezo-crystals the rule about the centre of symmetry dealt with in the foregoing pages has been found to be a good guide. Thanks to this rule attention has been particularly centered upon compounds having molecules lacking certain properties of symmetry, since it is to be expected that these compounds will form crystals having no centre of symmetry and thus yield a piezo-electric material. Synthetic piezo-crystals which have been developed and examined by Swiss and American investigators ⁵) among others are, for instance, ammonium dihydrogen phosphate (designated by the Bell Telephone Laboratories as ADP), dipotassium tartrate (DKT) and ethylene-diamine tartrate (EDT.) These crystals can be given the desired shapes fairly easily, they have a small internal damping and are not too readily dissociated (they have little or no crystal water). DKT and EDT possess piezo-electric constants 6 to 8 times as great as those of quartz. The ratio of the capacitances C_2/C_1 (see fig. 6) is 4 to 10 times smaller than that of quartz, so that they have wider ranges of resonance. Thanks to

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³) W. A. Wooster and N. Wooster, Nature 157, 405, 1946.

⁴) I. Franke and M. Huot de Longchamp, C. R. Acad. Sci. Paris **228**, 1136, 1949, No. 13.

⁵) See, i.a., W. Lüdy, Helv. Phys. Acta **12**, 279, 1939; W. P. Mason, New low-coefficient synthetic piezoelectric crystals for use in filters and oscillators, Proc. I.R.E. **35**, 1005-1012, 1947.

these properties the materials mentioned, in particular EDT, have already been applied on a rather large scale for transducers in carrier-telephony systems. Investigations are being continued, however, because the synthetic crystals so far produced cannot be regarded as equivalent substitutes for quartz. Even when various "temperature-independent" cuts are used their temperature coefficients are still 20 to 30 times, in the most favourable case still 5 times, as high as those of quartz; and for what may be the most important application of the piezo effect, viz. the stabilization of oscillators, it is just a high C_2/C_1 ratio that has to be aimed at, such as obtained with quartz.

Finally, mention should be made of the development of a piezo-electric material of an entirely different type, namely barium titanate. The peculiarity about this material is the fact that it is used in the polycrystalline form. With a polycrystalline material nothing is noticed of the normal piezo-electric effect because the differently orientated crystals neutralize each other's action. There is, therefore, in this case no question of a normal but rather of a so-called induced piezo-electricity. This effect is to be derived from the electro-striction mentioned in the beginning of this article and, consequently, is in principle present in every substance. The phenomenon of electro-striction implies that a deformation x takes place which is proportional to the square of an applied electric field:

$$x = a \cdot E^2$$

(E = electric field strength, a = material constant). When a field E_0 is applied a small change ΔE causes an extra deformation Δx proportional to ΔE, since

$$x + \Delta x = a(E_0 + \Delta E)^2 \approx aE_0^2 + 2aE_0 \cdot \Delta E,$$
$$\Delta x = 2aE_0 \cdot \Delta E.$$

Thus this is equivalent to a normal piezo-electric effect (actually the converse effect, but the direct effect then also occurs). With most substances this effect is extremely small. In the case of barium titanate, however, the constant a is enormously large, millions of times greater than that of other substances, whilst moreover this material is an "electret", that is to say, after removal of the external electric field the material retains a polarisation which can take the place of the external field E_0 [6]). Thanks to these properties practical use can be made of the phenomenon, and in fact crystal pick-ups made with barium titanate are already being manufactured on a rather large scale [7]). The advantage of this material is that it can be processed by ceramic means and thus can be given all sorts of shapes, which is not possible with mono-crystals. Nothing can be predicted at the moment as to how far the possibilities of application of this and similar materials will be extended in the course of time.

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[6]) These properties may be compared to those of ferromagnetic substances. They are also related to another peculiar property of barium titanate, viz. the exceptionally high value of its dielectric constant in a certain temperature range ($\varepsilon \geqq 10{,}000$).

[7]) L. Grant Hector and H. W. Karen, Electronics **21**, December 1948, pp. 94-96.

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R. A. Heising, Quartz crystals for electrical circuits. Their design and manufacture, Van Nostrand, New York 1946.

Summary. This article gives a brief review of the most important facts in the domain of piezo-electricity. In particular the relation is explained between the occurrence of this phenomenon and the symmetry of the crystal. The favourable properties of quartz for various applications of the piezo-electric effect are put forward. Finally mention is made of the search for new piezo-electric materials intended to be used partly as substitutes for quartz and partly for applications for which quartz is unsuitable. Polycrystalline barium titanate is also briefly referred to in this connection.

AN ARRANGEMENT FOR INDICATING PIEZO-ELECTRICITY OF CRYSTALS

by W. G. PERDOK*) and H. van SUCHTELEN.

621.317.36:537.228.1

The strong piezo-electric effect of a quartz crystal was discovered by the Curie brothers with comparatively simple means. To indicate the very weak effect of some species of crystals, of which only minute fragments may be available, more sensitive methods are needed. Applied electronics provides such methods and at the same time makes it easy to carry out extensive and systematic investigations.

There are two sides to the investigation into the piezo-electric behaviour of a substance. In the first place it is needed in the search for new, natural or synthetic materials for the practical application of the piezo-electric effect, the importance of which has been pointed out in the preceding article in this number [1]). In the second place information as to whether a substance is piezo-electric or not provides a guide when investigating the structure of a crystal, since a crystal can only be piezo-electric if it has no centre of symmetry, as likewise explained in the preceding article. Of the 32 classes to which a crystal can belong, the eleven classes with centre of symmetry (and also one of the 21 other classes) are at once eliminated when the crystal is found to be piezo-electric. Moreover, from details of the piezo-electric behaviour it is possible to draw farther-reaching conclusions regarding the crystal structure.

For both these investigations it is useful to have an instrument with which many different substances can be quickly tested for the presence of piezo-electric effect as a first orientation.

It was for this purpose that Giebe and Scheibe designed an apparatus in 1925, which has served as a basis for various modern instruments [2]). The device forming the subject of this article is also based upon the fundamental idea of the Giebe and Scheibe apparatus. The improvement compared with older designs lies in greater sensitivity, clearer indication and easier working [3]).

Principle of the method

Suppose a thin plate sawn from the rough material to be placed between two parallel flat electrodes and electrically connected parallel to a tuning circuit of an oscillator; see fig. 1. If the crystallographic axes in the plate are suitably orientated with respect to the plane of the electrodes then, in the case of a piezo-electric material, a coupling arises between the mechanical vibrations and the electrical oscillations, which can be described with the aid of the equivalent circuit for the crystal (fig. 2). In the larger part of the frequency range the crystal behaves as a capacitor with the capacitance C_2. In a certain, very narrow frequency range, however, a resonance phenomenon occurs, viz. a series resonance, whereby the impedances of L and C_1 neutralize each other, and (at a slightly higher frequency) also a parallel resonance where the impedance of L is equal and opposed to that of

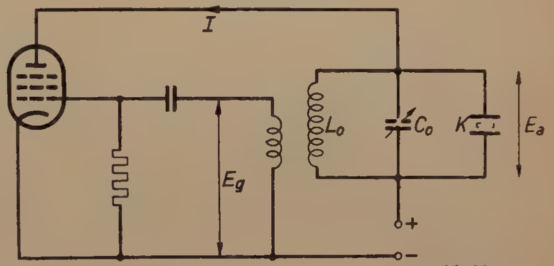

Fig. 1. Oscillator circuit with tuning circuit L_0-C_0 shunted by a crystal between two flat electrodes (K).

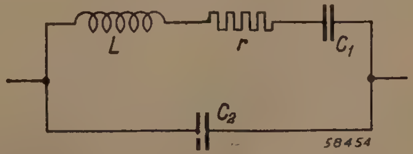

Fig. 2. Equivalent circuit of a vibrating piezo-electric crystal between two electrodes. The static capacitance C_2 is generally much greater than the capacitance C_1 representing the mechanical rigidity.

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*) Of the Crystallographic Institute of the Groningen University (Netherlands).

[1]) J. C. B. Missel, Piezo-electric materials, Philips Techn. Rev. **11**, 14 -150, 1949 (No. 5).

[2]) E. Giebe and A. Scheibe, Z. Phys. **33**, 760, 1925. Further, i.a. S. B. Elings and P. Terpstra, Z. Krist. **67**, 279, 1928, E. Burnstein, Rev. sci. Instr. **18**, 317, 1947, R. F. Stokes, Amer. Mineralogist **32**, 670, 1947.

[3]) The arrangement has been previously described in: W. G. Perdok and H. van Suchtelen, Appl. sci. Res. B1 195-204, 1948, (No. 3).

C_1 and C_2 (see the reactance diagram in fig. 7 of the preceding article). In the case of the series resonance it is as if the admittance of the small resistor r suddenly came to lie in its entirety parallel to the oscillator circuit. Thus this circuit becomes all at once strongly damped.

This behaviour can be taken as an indication that a crystal is piezo-electric. To determine whether this is so, the oscillator circuit is tuned by means of a variable capacitor to various frequencies in succession. When the frequency of the series resonance is reached the amplitude of oscillation suddenly drops, returning to its initial value immediately when the frequency is further changed. This drop in the amplitude is detected and made audible in a loudspeaker.

The resonance phenomenon occurs at different frequencies according to the dimensions of the plate and the orientation of the crystallographic axes with respect to the electrodes. Only with some rather sharply-defined orientations will the effect be quite clear. Now it is not practicable to have to saw every time from the crystal a plate with a different orientation and then scan the frequency range for each plate. Moreover, in many cases there would not even be sufficient material to do this, only some splinters or grains of the crystal being available. This, however, is no objection in principle. It is the essence of the Giebe and Scheibe method that a quantity of splinters of the sample to be tested can be placed between the parallel electrodes and that among these fragments of crystal there will always be some having a suitable orientation to exhibit the effect (hence the effect will generally be noticed at more than one frequency while passing through the range).

The space between the electrodes is, it is true, filled only for a small part by the piezo-electric crystal. The impedance of the crystal is measured, as it were, with the parallel connection and series connection of two constant capacitances (the non-reacting crystals and the air between the crystal and the electrodes). The dips in the impedance measured are therefore only very small, so that in order to indicate the sometimes very weak piezo-electric effects of all sorts of crystals the device has to answer very high demands as regards sensitivity.

Means of increasing the sensitivity

Small dips in the impedance between the electrodes will have a greater effect upon the oscillation according as the impedance of the tuning circuit itself is greater. This impedance is determined for a large part by the internal resistance of the oscillator valve connected to it in parallel. It is therefore advantageous to choose a valve having a high internal resistance, such as a pentode.

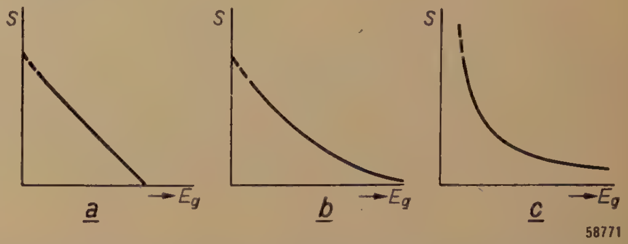

Fig. 3. The relation between the effective mutual conductance S and the alternating grid voltage E_g for different types of valves: *a*) linear, *b*) exponential, *c*) hyperbolic.

Another important feature of the valve to be used is the relation between the effective mutual conductance and the alternating grid voltage; see *fig. 3*. This can be explained as follows:

The effective mutual conductance S is defined by

$$I = S \cdot E_g \,, \qquad \ldots\ldots\ldots \qquad (1)$$

where E_g is the amplitude of the (sinusoidal) grid voltage and I the amplitude of the first harmonic of the anode current. Since the tuning circuit in the anode circuit always oscillates at its natural frequency, the anode impedance can simply be regarded as a resistance R. The anode current I induces at this resistor an alternating voltage

$$E_a = I \cdot R = S \cdot E_g \cdot R \,,$$

a fraction of which, $t \cdot E_a$, is fed back to the grid. The circuit will reach a stabilized oscillation when $t \cdot E_a = E_g$, thus when

$$tSR = 1. \qquad \ldots\ldots\ldots \qquad (2)$$

At given values of t and R the amplitude of the oscillation, or, to express it otherwise, the "working point" on the "S-E_g characteristic" (fig. 3), adjusts itself so that the mutual conductance answers the equation (2) [4]. A variation $\mathrm{d}R/R$ in the anode impedance, brought about when a crystal resonance is passed in the changing of the circuit frequency, will result in an equal change $\mathrm{d}S/S$ in the mutual conductance and thus a displacement of the working point. Now we use as indication for the state of oscillation of the circuit the grid voltage E_g (or the proportional alternating anode voltage $E_a = E_g/t$). It will then depend upon the shape of the characteristic in fig. 3 at what working point the greatest variation dE_g takes place for a given relative variation dS/S, or in other words at what strength of oscillation the most sensitive indication of the piezo-electric effect is obtained.

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[4] In the usual oscillator circuit according to fig. 1, when there is an alternating grid voltage of the amplitude E_g then, owing to grid rectification, a grid bias is automatically obtained which has practically the value E_g. It is due to this that the characteristics in fig. 3 always show a drop in S with increasing E_g. Only with such a variation is the condition according to equation (2) a stable state of equilibrium, the system always tending to revert to this state in the event of any deviations.

As oscillator valve we have chosen a variable-mu pentode (EF9), the relation S-E_g of which is approximately exponential. A simple calculation shows that with an exponential relation the variation dE_g referred to is independent of the working point. This is very useful in practice because the amplitude of oscillation will naturally vary gradually when a large frequency range is scanned; it would be an awkward complication if, in order to maintain the same sensitivity, the amplitude of the oscillation should have to be repeatedly readjusted, for instance by changing the feedback factor t.

The dips in the alternating anode voltage E_a taking place when passing the resonance frequencies of the crystals are made audible in a loudspeaker by means of a detector circuit, connected to the anode, and an A.F. amplifier.

Under the conditions described here it is advantageous to apply anode detection by means of an amplifying valve instead of the usual simple rectification by a diode. The grid of the amplifying valve is so heavily biased that anode current flows only during a part of each cycle of the alternating grid voltage (*fig. 4*). The amplitude of the anode-current pulses and thus also the mean anode current then depends upon the amplitude of the grid voltage. Thus a dip in the oscillator amplitude causes a corresponding dip in the mean voltage at a coupling resistor in the anode circuit of the detector valve. Thanks to the amplification by the detector valve, this voltage pulse is sufficient to be further amplified in a normal A.F. amplifier, for instance the A.F. part of a radio receiver.

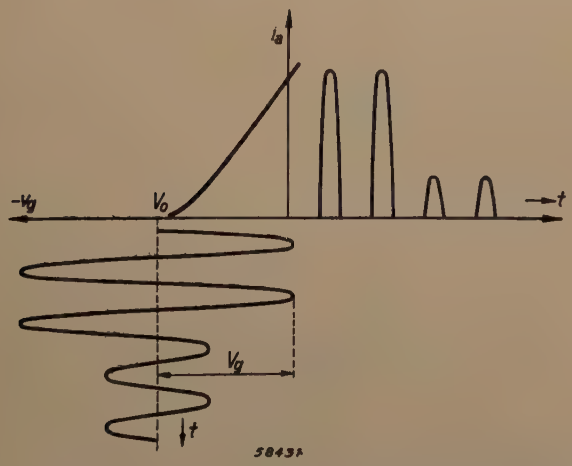

Fig. 4. Anode detection. A dip in the amplitude V_g of the alternating grid voltage is accompanied by a corresponding dip in the amplitude of the anode-current pulses, provided the grid bias V_0 remains constant.

The negative bias for this anode detector, which has to be matched to the oscillator amplitude, is most easily derived from that amplitude itself via a grid capacitor and leak. But of course the bias must not change during the dip in the oscillator amplitude that is to be indicated, for then the effect of the anode detection would for the greater part be lost. Therefore a grid capacitor and leak of a sufficiently high value to ensure a large time constant are used, so that only gradual changes of the oscillator amplitude will affect the grid bias.

In other developments of the Giebe and Scheibe principle there is no separate detector and anode detection takes place in the oscillator itself. In fact, the form of the grid voltage and the anode current of the oscillator valve are in principle identical to those of our detector valve as shown in fig. 4. When passing a crystal frequency one can therefore get directly a D.C. voltage surge at a series resistor in the anode circuit of the oscillator valve. But then we get the effect mentioned above where the automatic adjustment of the grid bias keeps the anode current impulses practically constant; the drop in amplitude of the grid voltage is then mainly manifested in a slight widening of the anode current impulses.

The greater sensitivity obtained with a separate detector is thus due in part to the very fact that the grid capacitor and leak can be given a high value, which is not possible with the oscillator valve without adversely affecting the functioning of the oscillator (cf. footnote [4]).

Wobbling tuning

When investigating different kinds of crystals it cannot of course be predicted in what frequency ranges resonances will be found. The oscillator is therefore fitted with exchangeable or variable coils each covering a certain frequency range to be scanned by means of a variable capacitor. As the capacitor is slowly turned a click will be heard in the loudspeaker as soon as a resonance frequency is reached. With a view to making such a point of resonance more noticeable, in earlier designs of apparatus it was recommended to wobble the knob of the capacitor to and fro while gradually turning it farther. Instead of one single click a number of clicks are then heard, but one has to make sure that these sounds are not caused by a "creaking" of the capacitor shaft.

This process has now been very much simplified by an automatic and continuous wobbling of the tuning brought about with the aid of a small vibrating capacitor. This capacitor, consisting of a fixed plate and a movable one, mounted on a leaf spring kept in vibration at a frequency of 100 c/s electromagnetically, is shunted across the tuning capacitor and causes the total tuning capacitance to vary periodically by a few pF. One can now steadily turn the tuning capacitor in one direction, and when a resonance is reached a rattling noise with the fundamental frequency of 100 c/s is heard in the loudspeaker.

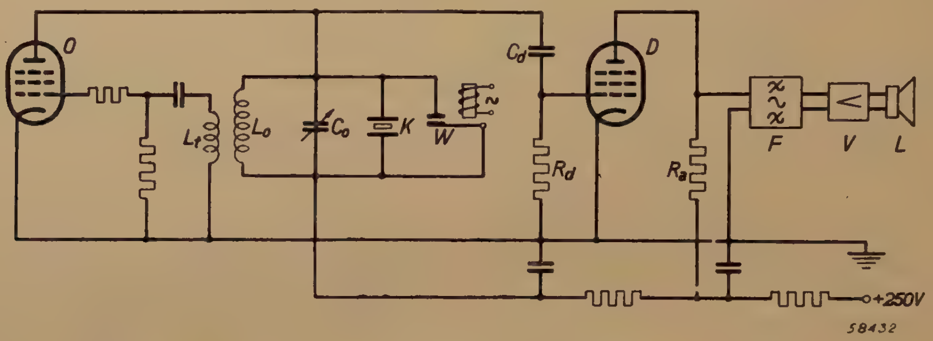

Fig. 5. Circuit diagram of the whole apparatus (somewhat simplified). *O* oscillator valve, L_0-C_0 tuning circuit, *K* crystal holder, *W* wobbling capacitor, *D* detector valve for anode detection, with coupling capacitor, C_d and grid-leak R_d. The signal to be made audible is taken from R_a. *F* band-pass filter for 600 c/s, *V* audio-frequency amplifier, *L* loudspeaker.

With the gradual change of the tuning capacitance also the impedance of the circuit gradually changes, but this is not noticed in the loudspeaker. However, the rapid periodical variation caused by the vibrating capacitor, although still bearing a much more continuous character than the dip when passing a resonance, can indeed be heard as a hum in the loudspeaker, particularly when the tuning capacitor is turned almost to its minimum. The effect sought might be more or less masked by this hum. To avoid this, the signal obtained after detection is passed through a relatively narrow band-pass filter tuned to the (somewhat arbitrarily chosen) frequency of 600 c/s, which strongly attenuates both lower and higher frequencies. Behind the filter the variation of the oscillation amplitude with 100 c/s, which variation is roughly sinusoidal, is practically unnoticeable, whilst also noise and other interferences are effectively suppressed, whereas the essential part of the rattling signal (caused by a number of higher harmonics of 100 c/s, e.g. 500, 600, 700 and 800 c/s) is passed through.

Practical construction

The whole of the circuit, based on the principles described above, is represented in *fig.* 5. Here the crystal holder and the wobbling capacitor with its drive are seen connected parallel to the tuning circuit. The oscillator valve is inductively fed back to the tuning coil. Actually there are five interchangeable tuning coils with their feed-back coils, covering a wavelength range from about 10 m to 1000 m.

The coupling capacitor C_d between the oscillator valve and the detector valve has the fairly high value of 50,000 pF. Thus with a grid-leak resistor of 2 MΩ the *RC*-time of the coupling is 0.1 sec., which is long enough to avoid any noticeable change in the grid bias of the detector when a resonance is reached.

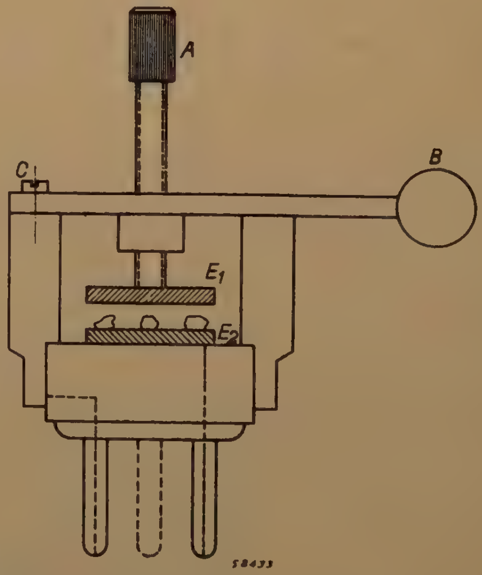

Fig. 6. Crystal holder with electrodes E_1 and E_2 mounted on a normal valve base (in this case with only two connecting pins in use). By turning the screw *A* the upper electrode E_1 is lowered until it is immediately above the crystals (it must not touch them). In this way the sensitivity is increased to the utmost. The holder is opened by turning aside the cover together with the upper electrode, the cover being rotatable about the vertical axis *C* by means of the knob *B*.

Former designs of apparatus working on the Giebe and Scheibe principle have all been built for battery supply, at least as regards the oscillator feed. This was necessary because otherwise the very weak clicking in the loudspeaker would have been drowned in the hum, in itself also weak, caused by the A.C. mains frequency of 50 c/s. With our set-up, owing to the measures taken for increasing the sensitivity, the indication in the loudspeaker is so clear that the hum can be ignored. The apparatus is therefore fed entirely from the mains, which of course makes it very much easier to use.

Finally, *fig. 6* gives an idea of the crystal holder, which is mounted on a valve base. It is very convenient to be able to remove this holder for cleaning and for refilling with samples.

This construction has proved to be sufficiently sensitive to be able to detect quite clearly the weakest piezo-electric effects, such as those of sodium santoninate and melinophane, which hitherto could only be detected with difficulty.

Summary. According to the Giebe and Scheibe method the piezo-electric effect of a crystal is indicated by placing a number of fragments of crystal between two flat electrodes and connecting the latter parallel to the tuning circuit of an oscillator. The dips in the oscillation amplitude when the natural frequencies of the crystals are reached are made audible in a loudspeaker.

By employing a variable-mu pentode as oscillator valve it has been possible to ensure reasonably large dips in the amplitude, whilst the sensitivity of this indication is also practically independent of the oscillation amplitude. Sensitivity is further increased by detecting the dips in the anode voltage of the oscillator with a separate amplifying valve. The tuning frequency of the oscillator is made to wobble over a small frequency range by means of a small vibrating capacitor. When a crystal frequency is reached, instead of a single click a rattling tone is heard in the loudspeaker, which is much less liable to escape notice.

A MODEL FOR STUDYING ELECTROMAGNETIC WAVES IN RECTANGULAR WAVE GUIDES

by K. S. KNOL and G. DIEMER.

538.566.5:621.392.26:621.317.35

Wave guides are metal tubes used for transmitting electromagnetic energy of very short wavelengths over short distances, for instance from one part of a transmitter to another. In the case of guides having a rectangular cross section, the wave phenomena occurring can be studied with the aid of a mechanical model in the form of a vibrating membrane. Such a model is not only of value as an ingenious means of demonstrating all sorts of properties of wave guides but it is also of immediate practical importance for the designing of wave guides, since it enables one to determine easily the effect of varying certain parameters.

The membrane as a means of studying electrostatic fields

A well-known means of studying two-dimensional electrostatic fields consists of a membrane in the form of a sheet of rubber stretched in a frame with a constant tension on which the boundary conditions of the electrostatic field being studied are imitated [1]). These conditions are imitated by imparting to the sheet of rubber, with the aid of rigid rods, constant deflections proportional to the potential of the electrodes of the electrostatic problem. Provided the deflections are small, the shape then assumed by the sheet (a minimum surface) corresponds to the potential variation sought, as shown in the article quoted in footnote [1]).

In that article it was found that the sum of the forces acting upon a surface element dxdy of the sheet equals

$$s\left(\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}\right)\mathrm{d}x\,\mathrm{d}y,$$

in which s is the tension in the sheet, h the deflection from the original (flat) state of equilibrium, and x and y are Cartesian co-ordinates in the plane of that state of equilibrium. In the new state of equilibrium this sum of forces is everywhere zero, so that one arrives at the differential equation of Laplace

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}=0. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

It is known that for the electrostatic potential V in free space the same equation holds, in the general case with three co-ordinates:

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=0.$$

When, however, one has to deal with a case where the potential is not dependent upon one of the co-ordinates, say upon z, so that the problem can be formulated with only two Cartesian co-ordinates, this equation becomes

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}=0,$$

from which it follows that the potential V is proportional to the deflection of the sheet h which satisfies equation (1).

In the article referred to, the use of this model for studying electron paths in radio valves is explained. It will now be shown that a somewhat modified form of this model lends itself to the study of electromagnetic waves; in this case the membrane is vibrated instead of being brought to a state of equilibrium.

The vibrating membrane as a means of studying electromagnetic waves

Vibrations of the membrane are governed by the same differential equation as electromagnetic waves in a space with conductive walls, again provided these waves can be formulated with only two Cartesian co-ordinates. This will be made clear from what follows:

When the membrane is in vibration, then at any arbitrary moment the sum of the forces acting upon a surface element dxdy is not zero but equal to the product of mass and acceleration of the element. If ϱ is the mass of the sheet per unit area and t the time, then one finds from the equality just mentioned, after dividing by dxdy,

$$s\left(\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}\right)=\varrho\,\frac{\partial^2 h}{\partial t^2}.$$

Putting $s/\varrho = v^2$ and replacing y by z to agree

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1) P. H. J. A. Kleynen, The motion of an electron in two-dimensional electrostatic fields, Philips Techn. Rev. 2, 338-345, 1937.

with the article in this journal 2) on electromagnetic waves, we arrive at

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial z^2}-\frac{1}{v^2}\frac{\partial^2 h}{\partial t^2}=0. \quad . \quad . \quad . \quad (2)$$

This is the well-known wave equation. Solutions of this equation are progressive waves, for example of the form:

$$h=H\sin\omega\left(t\pm\frac{z}{v}\right)$$

($H =$ amplitude, $\omega =$ angular frequency), as can be seen by substitution in equation (2). The term v is the velocity at which the wave is propagated across the membrane.

In the case of a plane electromagnetic wave in an unbounded homogeneous and isotropic medium (with magnetic permeability μ, dielectric constant ε and electric conductivity σ) for the components H_x, H_y, H_z, E_x, E_y, E_z of the magnetic and electric field strength there are six simultaneous differential equations 3). In two of these there occurs, for instance, the component E_y, viz:

$$\frac{\partial E_y}{\partial z}=\mu\frac{\partial H_x}{\partial t}$$

and

$$\frac{\partial H_x}{\partial z}=\sigma E_y+\varepsilon\frac{\partial E_y}{\partial t}.$$

Eliminating H_x and putting $\sigma = 0$ (non-conducting medium) and $\varepsilon\mu = 1/v^2$, then

$$\frac{\partial^2 E_y}{\partial z^2}-\frac{1}{v^2}\frac{\partial^2 E_y}{\partial t^2}=0. \quad . \quad . \quad . \quad (3)$$

This is the equation for the component E_y of a plane wave travelling in the z-direction with a velocity of propagation v, which in vacuum is equal to $c \approx 3{\cdot}10^8$ m/sec. Analogous equations hold for the other components of the electric field strength E and likewise for the three components of the magnetic field strength H.

By a linear superposition of waves travelling in the x- and z-directions, for a two-dimensional case where the quantities do not depend upon y, we arrive at the equation

$$\frac{\partial^2 F}{\partial x^2}+\frac{\partial^2 F}{\partial z^2}-\frac{1}{v^2}\frac{\partial^2 F}{\partial t^2}=0, \quad . \quad . \quad . \quad (4)$$

in which F may represent any component of E or H. This wave equation is quite analogous to the equation (2) found above for the vibrating membrane. Thus the term F corresponds to the deflection h of the membrane. This means that the vibrating membrane can serve as a model for studying electromagnetic waves in which the field quantities can be expressed in terms of only two Cartesian co-ordinates, since only two such co-ordinates are available on the membrane. This condition is satisfied in the case of transverse electric (TE) waves 4), a wave form with which one is almost exclusively concerned in rectangular wave guides.

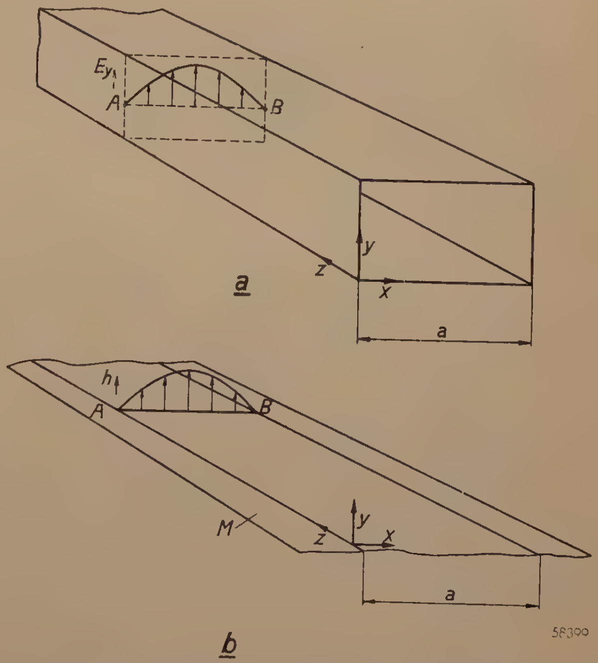

Fig. 1. *a*) Rectangular wave guide in which a transverse electric wave is propagated. The only component E_y of the electric field strength along a line AB (parallel to the x axis) at a given moment, in the case of the simplest form, follows a sine AB (the straight line AB runs parallel to the x-axis), independently of the distance y between AB and the x-z plane. *b*) Corresponding to the field strength E_y in the wave guide is the deflection h of a transverse vibrating membrane M clamped along two parallel straight lines.

Fig. 1a is a diagrammatic representation of a rectangular wave guide, with the co-ordinate axes indicated. For the components of the electric field strength in a TE-wave, according to the equations (29) and (30) of the article quoted in footnote 2), we have

$$\left.\begin{aligned}&E_x=E_z=0\,, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .\\ &E_y=E_{\max}\sin\frac{m\pi x}{a}\cdot\exp j\omega\left\{t-\frac{z}{c}\sqrt{1-\left(\frac{m\lambda}{2a}\right)^2}\right\},\end{aligned}\right\}\quad(5)$$

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2) W. Opechowski, Electromagnetic waves in wave guides, Philips Techn. Rev. 10, 13-25, 1948 (No. 1).

3) See e.g. (5a)... (6c) in the article (page 16) quoted in footnote 2).

4) See, e.g., the article (page 22) quoted in footnote 2).

in which $E_{\max}$ is the maximum amplitude of E_y, a the width of the wave guide, ω the angular frequency, $\lambda = 2\pi c/\omega$ the free-space wavelength and $m = 1, 2, 3, \ldots$ In a given cross section ($z =$ constant) and at a given time t it thus follows that the only component, E_y, depends upon x according to sin $(m\pi x/a)$. Since E_y is independent of y this is true, with the same proportionality factor, for any height y above the plane x-z. In fig. 1*a* the simplest wave form is represented, namely that for $m = 1$, whereby the width a of the wave guide is covered by one half-cycle of the sine representing E_y as a function of x. Both with this mode and with that where $m = 2, 3, \ldots$ one finds $E_y = 0$ for $x = 0$ and for $x = a$. This is in agreement with the boundary condition which requires that the component of the electric field in a plane of a vertical wall of the wave guide is everywhere zero.

Corresponding boundary conditions have to be satisfied with the vibrating membrane. That is to say, the amplitude of the membrane must be kept zero along two parallel lines (fig. 1*b*). This requirement is easily met by clamping the membrane along such lines, as we shall see when we come to the description of the apparatus.

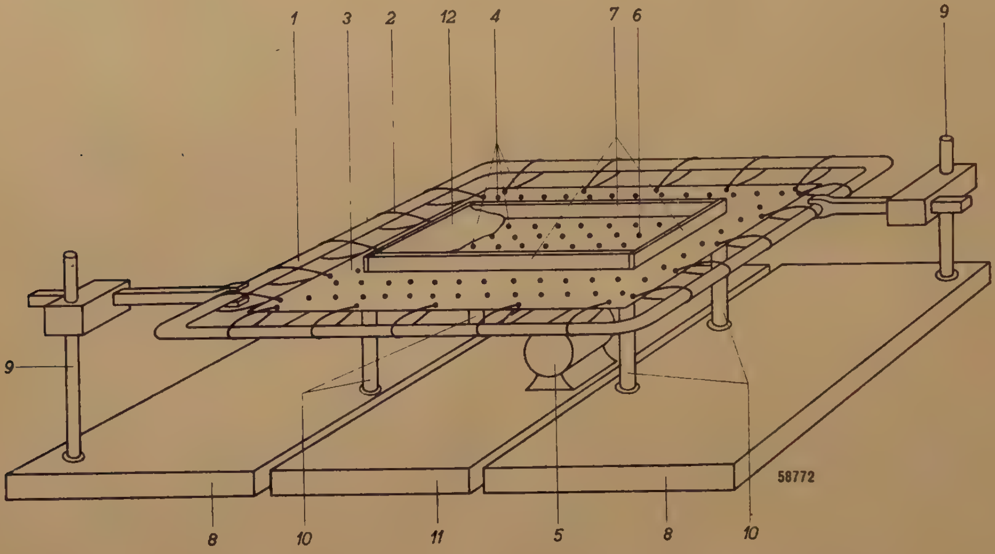

Fig. 2. Sketch of the mechanical model for studying two-dimensional waves. *1* = frame of steel tubes on which a sheet of rubber *3* is stretched by cords *2*, the rubber thus serving as a membrane. *4* = white dots making the movement of the membrane easily visible. *5* = motor driving the membrane at the point *6*. *7* = plane-ground bars (a second set is underneath the membrane) between which the membrane is clamped according to the boundaries of the wave guide being imitated. *8* = tables on which the frame *1* (via the rods *9*) and the bars *7* (via the stands *10*) rest. *11* = heavy iron table with vibration-free mounting, carrying the motor *5*. *12* = wedge-shaped pad of cotton-wool closing the wave guide and making it free of reflections.

Two cases should be mentioned in which the waves cannot be described with two Cartesian space coordinates and consequently cannot be studied with the aid of this model.

In the first place there are the transverse magnetic (TM) waves in a guide of rectangular cross section. As explained in the article quoted in footnote [2]) (p. 23), in a rectangular wave guide only certain TM-waves can be propagated, which are of a much more complex structure than the TE-waves according to eq. (5). This implies, inter alia, that in the case of such TM-waves the field components depend not only upon x and z but also upon y.

Neither is the model suitable for studying circular wave guides. In a circular guide, where cylindrical co-ordinates (r, φ, z) will obviously be used, wave forms are possible in which the field components are independent of φ. This, it is true, leaves only two co-ordinates (r, z), but these do not correspond to the Cartesian co-ordinates on the membrane.

Construction of the model with vibrating membrane

Fig. 2 is a sketch of the set-up arranged by us. A sheet of rubber is stretched in a strong frame with the aid of cords. The strain in all directions is about 10 %. Plane-ground bars representing the sides of the wave guide hold the rubber tight top and bottom. The "drive" is underneath the rubber at the place denoted by the figure *6*: by means of the motor *5* and an eccentric, a vertical rod is moved up and down; between the sheet and the rod is a piece of sponge-rubber. (Owing to the eccentric the movement of the rod is not absolutely sinusoidal, but the harmonics are not troublesome.) The motor stands on a separate, heavy, iron table (11) resting upon rubber blocks, thus avoiding parasitic couplings between the motor and the membrane. The motor is a direct-current motor, with its speed and thus the frequency of the membrane controlled by means of the armature voltage.

At the two short sides of the rectangle formed by the bars *7* reflections can take place just as would be the case in a wave guide closed by a conducting plate. Reflections at the side nearest to the driving point *6* are not troublesome if that point is suitably chosen; the result at some distance to the left of *6* (in fig. 2) is then always a wave coming from the right. If a wave guide is to be imitated in which, as is normally the case, the energy is consumed at the other end, then one must avoid reflection at the side of the membrane removed from the driving point. A suitable means of absorption has been found to consist in a pad of cotton-wool laid upon the membrane (*12* in fig. 2) with the thin end directed towards the driving point.

We now come to the practical question as to how the waves on the membrane can best be observed and their amplitude measured.

The oldest method is that of Makinson⁵), whereby fine sand is scattered over the horizontal membrane. As in the well-known Chladni sound figures, the grains of sand roll away from the antinodes and ultimately collect near the nodal lines. This method, however, gives only a qualitative picture and is not suitable for investigating progressive waves, while furthermore there is the practical objection that in course of time all the sand moves towards the walls (the only places continuing to be absolutely at rest). For these reasons, therefore, we tried to find better solutions and actually found them in the two following methods:

1) A rectangular co-ordinate system of white dots was painted on the sheet of rubber (see fig. 2). As the sheet vibrates, those dots which are not on nodal lines look like small rods whose length is proportional to the deflection. This gives a much more graphical picture than figures in sand. Moreover, the vibrating membrane can be photographed and by using a sufficiently long exposure and measuring the small white lines produced by the vibrating dots one can arrive at more or less quantitative results.

A valuable aid in observing the waves on the membrane is stroboscopic exposure⁶) at a frequency differing slightly from the frequency at which the membrane vibrates.

2) The amplitude at any point of the sheet of rubber can be more accurately determined by placing over that point a minute metal leaf spring mounted via an insulating cylinder on a micrometer screw at a short distance from the end of the screw (*fig. 3*). The screw is so adjusted that the vibrating sheet of rubber just presses the spring up against the end of the screw, thereby making and breaking an electrical contact, which can be made audible with the aid of an amplifier and a loudspeaker. The position of the micrometer screw in which contact is just made is a measure of the amplitude at that particular spot.

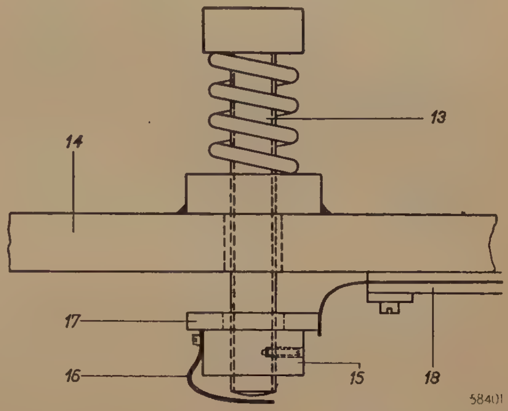

Fig. 3. Device for measuring the amplitude of the membrane at any point. *13* = micrometer screw. *13* is screwed into a nut soldered onto the plane-ground bar *14*, which slides along the bars 7 (see fig. 2). *15* = insulating cylinder, *16* = leaf spring, *17* = contact ring, *18* = wire for current feeding. The screw is placed over the point where the amplitude is to be measured and so adjusted that the vibrating membrane periodically touches the spring and just causes it to make contact with the screw, this being made audible by means of a signalling arrangement.

This latter method is particularly useful for determining the standing-wave ratio, i.e. the ratio of the maximum and minimum amplitudes along the axis of the wave guide or, respectively, of the membrane. This ratio is a measure of the reflection taking place in the wave guide⁷).

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⁵) R. E. B. Makinson, A mechanical analogy for transverse electric waves in a guide of rectangular section, J. sci. Instr. **24**, 189-190, 1947.

⁶) The stroboscope used (GM 5500) is described in Philips Techn. Rev. **8**, 25-32, 1946.

⁷) For Lecher systems (transmission lines) the standing-wave ratio — though this term was not introduced there — was mentioned implicitly in the article by J. M. van Hofweegen: Impedance measurements with a non-tuned Lecher system, Philips Techn. Rev. **8**, 278-286, 1946, in particular in fig. 2 and eq. (20).

Some properties of rectangular wave guides that can be demonstrated with the membrane.

Critical wavelength

From the equation (5) given above for the component E_y of a TE-wave an important property of rectangular wave guides can be deduced. For wavelengths which are less than a critical wavelength $\lambda_{cr} = 2a/m$ these guides behave quite differently from the way they do for wavelengths greater than λ_{cr}. Only for $\lambda \leqq \lambda_{cr}$ is $\sqrt{1-(m\lambda/2a)^2}$ real and thus the exponent purely imaginary, which corresponds to a wave travelling in the z-direction [8]. If on the other hand $\lambda > \lambda_{cr}$ then the exponent is complex, i.e. the amplitude of E_y diminishes exponentially in the z-direction. In practice use is made of both these possibilities.

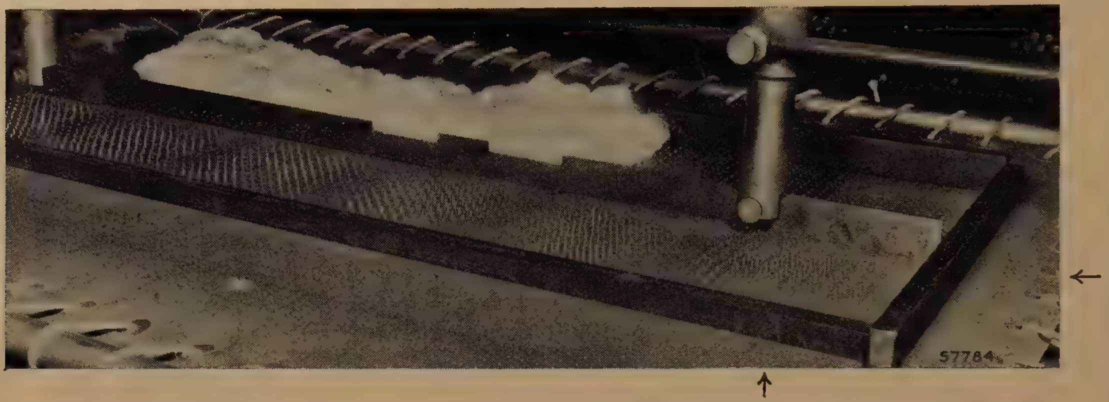

Fig. 4. Photograph of the membrane driven at a frequency higher than the critical frequency ($\lambda < \lambda_{cr}$). The arrows indicate the point at which the membrane is driven. Owing to reflection at the left-hand end (not visible in the picture) standing waves arise, the nodes and antinodes of which are clearly seen. In the background is a pad of cotton-wool which may be used for damping.

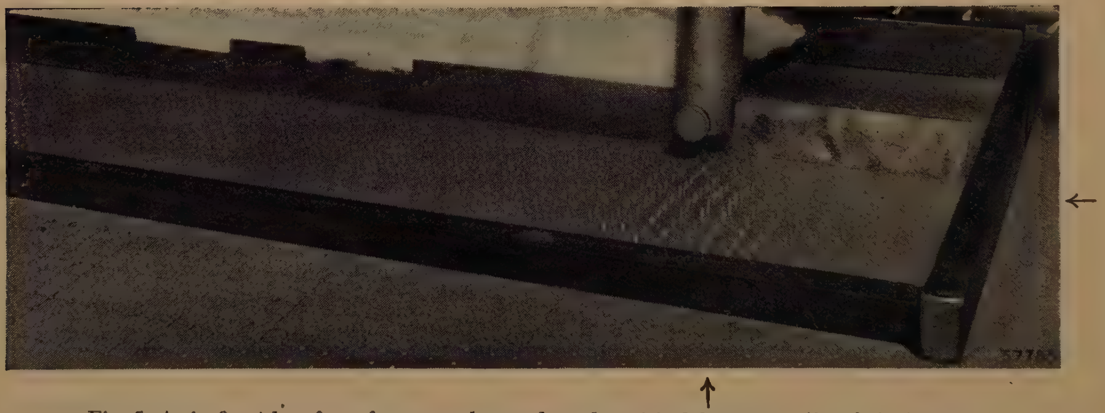

Fig. 5. As in fig. 4 but for a frequency lower than the critical frequency ($\lambda > \lambda_{cr}$). A wave cannot penetrate into the "wave guide" in this case; to the left of the driving point the amplitude rapidly decreases.

Where a wave guide has to transmit energy it should be so dimensioned that $\lambda < \lambda_{cr}$. The case where $\lambda > \lambda_{cr}$ is found in wave guides serving as attenuators, in which the change in field strength between two points along the axis of the guide is accurately known as a function of the distance (z) between them.

Figs 4 and 5 give two photographs of the membrane demonstrating this property. In fig. 4 the frequency has been so chosen that $\lambda < \lambda_{cr}$. It will be seen that a wave phenomenon occurs over the entire length of the "wave guide". The membrane was blocked at the end with a rigid rod (not visible in the photograph) and therefore corresponded to a wave guide closed by a conducting plate. The reflection gave rise to standing waves and the nodes and antinodes are clearly seen in fig. 4. It will also be seen that there is only one half wave across the width (due to the manner of drive), so that $m = 1$, just as in fig. 1.

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[8] In the article quoted in footnote [2]), page 24, left-hand column, third line from formula (31), $\lambda > \lambda_{cr}$ should be replaced by $\lambda < \lambda_{cr}$.

Fig. 5 relates to a lower frequency where $\lambda > \lambda_{cr}$. It is seen that the amplitude is practically zero even at a short distance to the left of the driving point.

From $\lambda_{cr} = 2a/m = v/f_{cr}$, where f_{cr} is the critical frequency, it follows that $f_{cr} = mv/2a$. The velocity of propagation v in the sheet of rubber is about 10 m/sec, so that with a width $a = 0.25$ m and the simplest mode ($m = 1$) for the membrane $f_{cr} \approx 20$ per/sec, $\lambda_{cr} = 0.50$ m.

Directional coupler

The second test to be described is on a directional coupler. This is illustrated in *fig. 6*, where A represents a wave guide (or in this case a membrane) driven at one end, at C, by $\lambda < \lambda_{cr}$. At the other end (W_A), as we shall first assume, the energy of the waves is absorbed, so that progressive waves are maintained. Side by side with this main wave guide is an auxiliary wave guide (B) coupled with A by two or more holes through which energy is transmitted from A to B. B is closed with a non-reflecting termination at the same end as A. In general, therefore, vibrations set up in B are propagated in both directions. If, however, the centre-to-centre distance of the holes is just $\lambda/4$, then in B the only possible direction of propagation is the same as in A (hence the name directional coupler). The reason for this is that two waves are propagated in the opposite direction in B. Since these are derived from two holes $\lambda/4$ apart, there is a phase difference of $2 \times \lambda/4$, i.e. 180°, and the waves cancel. According to the diagram in fig. 6, therefore, in the auxiliary wave guide there are only waves travelling to the left of the holes, and these are absorbed at W_B. To the right of the holes there is no energy at all, so that a detector placed at D does not indicate.

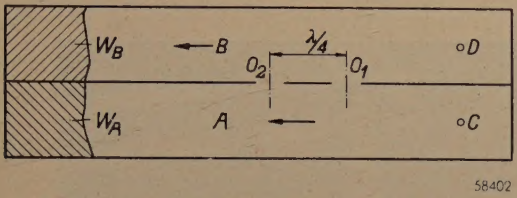

Fig. 6. Main wave guide A and auxiliary wave guide B, coupled via two holes (O_1, O_2) at a distance $\lambda/4$ (directional coupler). C = driving point, D = detector. W_A, W_B = damping material. In B waves are propagated only in the same direction as in A. When the absorption at W_A is absolute then in A waves travel only from C to W_A and in B only from the holes towards W_B; D receives no energy. If the absorption at W_A is not complete then in A waves also travel from W_A to C, setting up waves in B travelling from the holes in the direction of D; in that case the detector indicates.

The situation is different, however, when some reflection takes place in the main wave guide at W_A. In this wave guide and consequently in the auxiliary wave guide A too, there are then also waves travelling from left to right. In that case the detector D does indicate, and the quantity detected is a measure for the reflection at W_A. Thus a directional coupler can be used for investigating whether the energy in a wave guide does actually travel exclusively in the desired direction.

Figs 7 and 8 show how the action of a directional coupler can be demonstrated with the aid of the membrane. For this purpose the membrane is divided by clamping bars into two sections, that in the front, which is driven, corresponding to the main wave guide A (fig. 6) and that at the back corresponding to the auxiliary wave guide B. The coupling between the two sections is formed by two holes in the common intermediate bar between the sections. The frequency is so chosen that the centre-to-centre distance of the holes is $\lambda/4$. Absorption at the end of the second section is obtained with a pad of cotton-wool.

In the case depicted in fig. 7 a pad of cotton-wool was likewise laid on the membrane at the end of the section representing the main wave guide, so that there were only waves travelling from right to left. It is seen that there are also waves in the "auxiliary wave guide", but only to the left of the holes; to the right the membrane remains at rest. When, however, the absorbent material is removed from the front section then, owing to the reflection at the closing bar, waves are also propagated from left to right, and similarly in the second section. Consequently the right-hand part of the second section is set in motion, as can be seen in fig. 8. At the same time nodal lines are observed in the first section, so that standing waves arise as a consequence of reflection. (Between the driving point and openings a travelling wave, energizing the second section, is superposed upon these standing waves.)

Bevelling the bend in a wave guide

Where wave guides are used it is not usually possible to manage with simple straight pipes and one or more bends are unavoidable. This, however, calls for some care so as to avoid as far as possible the reflection of energy in the bend. In the case of a rectangular wave guide the simplest solution, from the constructional point of view, is to make a bend bevelled off on the outside. The question then arises as to what is the most favourable value of the parameter d/d_0 (see *fig. 9*).

A problem such as this can be solved very well with the aid of the membrane, since in the model

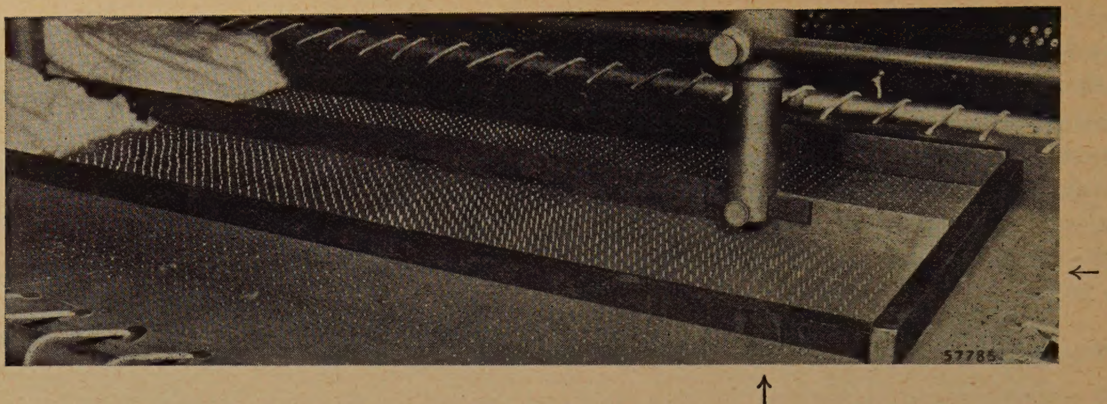

Fig. 7. Directional coupler imitated on the membrane. The front section corresponds to *A* (fig. 6) and the back one to *B*. In the intermediate wall are the two holes providing the coupling. To the left are the damping pads. Owing to the damping at the end of the front section the waves in this section travel from right to left only. Those in the second section travel in the same direction to the left of the holes. To the right (at *D* in fig. 6) the rear section remains at rest.

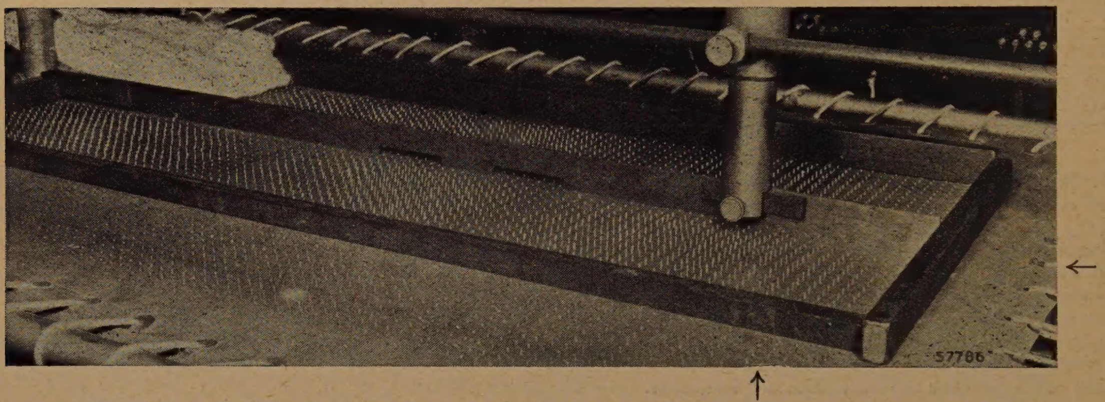

Fig. 8. As in fig. 7 but with no absorption in the front section. Standing waves now occur in the front section and in the rear section to the right of the holes. At the point *D* (fig. 6) a fairly strong vibration is set up; the amplitude is a measure of the reflection at the end of the front section.

the value of the parameter can easily be varied and the result more easily observed than is possible in an actual wave guide.

On the membrane we have imitated the shape of a wave guide bent at an angle of 90 degrees. With the aid of the aforementioned contact spring and micrometer screw the standing-wave ratio r has been measured, as a function of $d/d_0 = d/a\sqrt{2}$, of the wave patterns formed between the point of drive and the bend at different frequencies. The free end of the "wave guide" was closed with a non-reflecting termination.

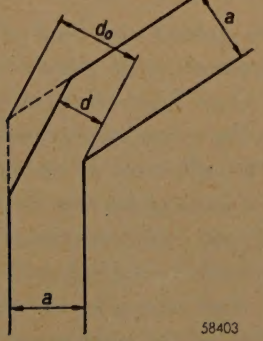

Fig. 9. When a wave guide has to be bent the bend should be bevelled off to prevent reflection, the ratio d/d_0 being given a certain optimum value.

In *fig. 10* $1/r$ is plotted as a function of d/d_0 for three frequencies. The smaller the standing-wave ratio — thus the greater $1/r$ — the less reflection takes place and thus the less disturbing is the bend in the wave guide. As may be seen from fig. 10, the optimum value of d/d_0 lies between 0.6 and 0.7, in agreement with the value found by entirely different means [9]).

The foregoing examples will have illustrated clearly enough that a mechanical model such as described here may be of great value for studying wave phenomena in rectangular wave guides. Naturally this also applies for cavity resonators and other configurations, provided the phenomena can be described with two Cartesian co-ordinates.

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[9]) See, e.g., G. L. Ragan, Microwave transmission circuits, Massachusetts Institute of Technology, New York 1948, page 207.

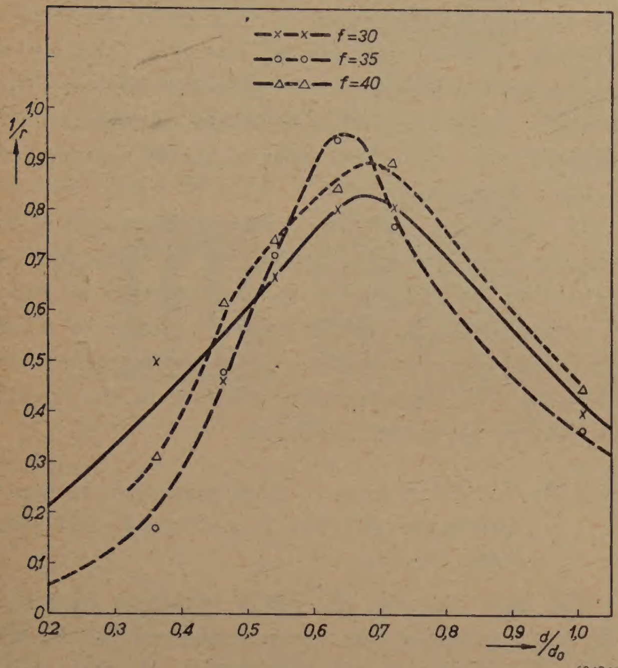

Fig. 10. On the membrane a wave guide bent 90 degrees is imitated and the standing-wave ratio r (in the absence of reflection this equals 1) is measured as a function of d/d_0 (fig. 9). The curves are related to the frequencies 30, 35 and 40 c/s. The optimum value of d/d_0 lies between 0.6 and 0.7.

It seems likely to us that electrical or mechanical vibrations in non-homogeneous media can also be investigated with the aid of such a model. The inhomogeneities of the medium would then have to be imitated in the model by varying the tension in the membrane and/or its density according to a certain function of the co-ordinates x and z. Many cases where calculations are too difficult could then be investigated by this means.

Summary. A mechanical model is described with which the properties of rectangular wave guides can be studied. It consists of a membrane in the form of a sheet of rubber stretched on a frame and caused to vibrate in a suitable manner. By clamping the sheet between flat bars, boundary conditions can be created corresponding to those of the wave guide. Absorption and reflection at the end of the wave guide can be imitated on the membrane by laying cotton wool on it and by clamping it. The membrane is driven by a direct-current motor via an eccentric. The movement of the membrane can easily be followed visually (especially when applying stroboscopic exposure) by the use of a network of white dots painted on the membrane. The amplitude of the vibration at any point can be measured with the aid of a simple instrument. With this model one can demonstrate and investigate the phenomenon of critical wavelength, the properties of a directional coupler and the most favourable bevelling of a bend in a wave guide. The value of the model described lies especially in its pictorial presentation and the ease with which various parameters can be altered. In principle it is also possible to make the model suitable for studying the propagation of waves in non-homogeneous media.

ABSTRACTS OF RECENT SCIENTIFIC PUBLICATION OF THE N.V. PHILIPS' GLOEILAMPENFABRIEKEN

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R 105: G. Diemer and K. S. Knol: Frequency conversion by phase variation (Philips Res. Rep. 4, 161-167, 1949, No. 3).

An improvement of Herold's method of phase reversal resulting in a still higher value of the conversion transconductance is studied theoretically. It is proved that the conversion transconductance can equal the maximum value of the high-frequency transconductance. This improvement by a factor $\pi/2$ in conversion transconductance over Herold's method is achieved by varying in a suitable way the phase of the transconductance with time. This is theoretically the highest value of the conversion transconductance that can be reached in any design. The theory has been corroborated by experiments with a tube for low frequencies.

R 106: A. van der Ziel and K. S. Knol: On the power gain and the bandwidth of feedback amplifier stages (Philips Res. Rep. 4, 168-178, 1949, No. 3).

The influence of feedback upon the power gain of an amplifier stage is discussed and it is shown that when for a given value of the feedback the limit of stability is reached, the power gain remains finite. The formulae are applied to the case of a grounded-grid stage and it is shown that rather small output losses may already give rise to a considerable decrease in gain. The influence of feedback upon the band width of an amplifier stage is also discussed. The results are again applied to a grounded-grid triode stage. The product of the band width B and the power gain g is plotted against g (with the coupling of the output circuit to the load as a parameter) for various values of the feedback capacitance and the output losses. It is shown that an amplifier stage having such an amount of feedback that it can just give oscillations for proper output coupling can only give a very high values of the power gain g for very small band widths. A considerable improvement in bandwidth is then possible by increasing the amount of feedback.

R 107: C. J. Bouwkamp: On the effective length of a linear transmitting antenna (Philips Res. Rep. 4, 179-188, 1949, No. 3).

The concept of effective length, proposed by King for the cylindrical, centre-driven antenna is extended. The new definition may be applied to any current distribution in the antenna. As an illustrative example a top-loaded antenna is discussed in detail.

R 108: H. Bremmer: Some remarks on ionospheric double refraction, II (Philips Res. Rep. 4, 189-205, 1949, No. 3).

Maxwell's equations for plane-wave propagation through a stratified anisotropic medium are reduced to a system of four ordinary first-order equations. These equations are very suitable for the derivation of W.K.B. approximations. The general theory given is worked out in detail for the special case of a stratified isotropic medium. In the last section the computation of field strengths in the case of ionospheric reflections is discussed (see these abstracts, No. **1851**).

R 109: J. A. Haringx: On highly compressible helical springs and rubber rods and their application for vibration-free mountings, III (Philips Res. Rep. 4, 206-220, 1949, No. 3).

This paper (see Nos **R 91** and **R 97**) deals with the elastic stability and the lateral rigidity of solid rubber rods under axial compression. The various calculations are based upon equations derived in the preceding papers for the elastic prismatic rod replacing the helical spring, though here some additional difficulties arise from the effect on the rigidity against shearing, caused by the inevitable vaulting of the initially flat normal cross sections of the rod. This effect, however, is of minor importance for rods of circular or rectangular cross section, so that in these cases a solution of the respective problems can be given. As to the buckling phenomenon a satisfactory agreement is found between the results of the present calculation and those of experiments by Kosten. The minor discrepancies occurring are probably due to the limited validity of the formulae introduced to describe the elastic behaviour of highly compressed rubber material.

If the rubber rods are vulcanized or affixed in some other way to metal pieces, at their ends a thin layer of material is to be regarded as fully incompressible, though it must be borne in mind that this layer retains its elasticity in respect of shearing. The corresponding effect upon the lateral rigidity is taken into account.

R 110: W. Elenbaas: High-pressure rare-gas discharges (Philips Res. Rep. 4, 221-231, 1949, No. 3).

The light output, the total radiation and the electrical gradient have been measured for A, Kr and Xe high-pressure discharges of the wall-stabilized type. The characteristics of these discharges have been calculated assuming the discharges to be in temperature equilibrium and the radiation to be mainly a recombination continuum. The light output, the total radiation and the electrical gradient have been derived assuming a constant energy per unit of frequency (as calculated by Unsöld) in the frequency band from $\nu = 0$ to $\nu = \nu_m = eV_i/h$. In the calculation of the electrical gradient the influence of the positive ions on the mobility of the electrons has to be considered. The influence of the atoms and that of the positive ions have been experimentally separated and the magnitude of both has been found in accordance with the calculations.

R 111: P. Cornelius: Proposals and recommendations concerning the definitions and units of electromagnetic quantities (Philips Res. Rep. 4, 232-237, 1949, No. 3).

To complete the arguments of a previous article (see No. **R 103**) the writer ventures to bring forward a set of proposals by which, in his opinion, many questions still in dispute may be finally settled.